CEE 307

STRUCTURAL MECHANICS I

- **Course code:** CEE 307
- **Course title:** Structural Mechanics I
- **Course unit load:** 2 Units (E: LH 45)
- **Series titles:**

1. Fundamentals of Statical Determinacy in Structures
2. Analysis of Statically Determinate Trusses
3. Shear Forces, Bending Moments, Slopes and Deflections in Beams and Portal Frames
4. Influence Lines and Applications to Moving Loads
5. Deflection Theorems and Introduction to Statically Indeterminate Structures

Proposed Outline for Series 1

- **Part 1.1 Concept of Statical Determinacy**
  - 1.1.1 Definition of statical determinacy
  - 1.1.2 Conditions for equilibrium in structures
  - 1.1.3 Classification of structures: determinate and indeterminate
- **Part 1.2 Types of Statically Determinate Structures**
  - 1.2.1 Beams
  - 1.2.2 Trusses
  - 1.2.3 Frames
- **Part 1.3 Importance and Applications of Statical Determinacy**
  - 1.3.1 Practical relevance in structural analysis
  - 1.3.2 Advantages and limitations of determinate structures
  - 1.3.3 Transition to indeterminate structures

3. Review of Outline Against Guidelines

- The outline contains **3 main Parts**, which is acceptable for a **2-credit course** (2 to 3 Parts required).
- Each Part is logically structured, progressing from definition and theory, to classification, and finally to applications.
- No merging or splitting is necessary, as the balance is appropriate.

Introduction

When engineers design bridges, buildings, or even simple roof trusses, one of the first questions they must ask is whether the structure can be analysed using only the basic equations of equilibrium. This property, known as **statical determinacy**, is fundamental because it determines whether the forces within a structure can be calculated directly without resorting to more complex methods. Understanding this concept provides the foundation for all subsequent analysis in structural mechanics and ensures that learners can confidently approach beams, trusses, and frames later in the course.

The aim of this Series is to introduce you to the principles of statical determinacy and to equip you with the ability to classify structures as determinate or indeterminate, while appreciating the importance of this distinction in engineering practice.

We shall commence this Series by examining the concept of statical determinacy, including its definition, conditions for equilibrium, and the classification of structures. Next, we will move on to explore the different types of statically determinate structures such as beams, trusses, and frames, highlighting their unique characteristics. Finally, we will wrap up by considering the importance and applications of statical determinacy, discussing its practical relevance, advantages, limitations, and how it leads into the study of indeterminate structures.

Series Learning Outcomes

Upon completing this Series, you should be able to:

- **SLO 1.1** define the concept of **statical determinacy** and state the conditions required for equilibrium in structures,
- **SLO 1.2** classify structures as **determinate** or **indeterminate** based on their equilibrium characteristics,
- **SLO 1.3** identify and describe the main types of statically determinate structures including beams, trusses, and frames,
- **SLO 1.4** explain the practical importance of statical determinacy in structural analysis and design, and
- **SLO 1.5** evaluate the advantages and limitations of determinate structures, while recognising their role as a foundation for studying indeterminate structures.

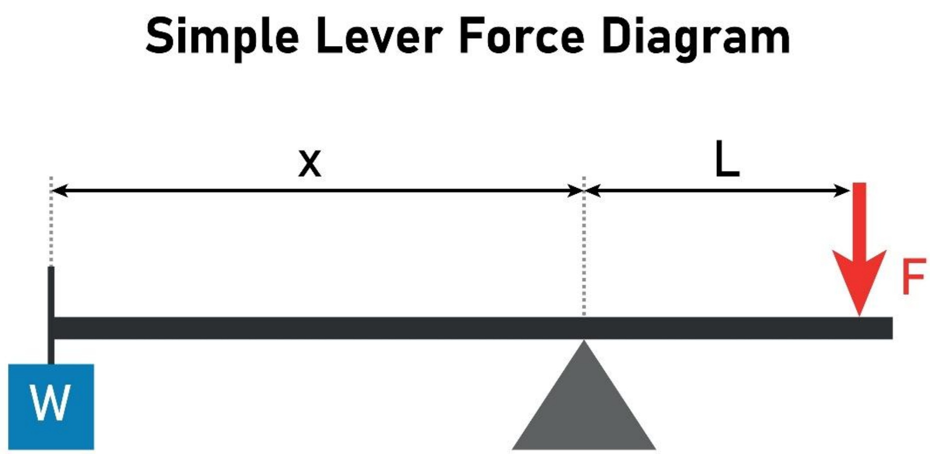
1.1 Concept Of Statical Determinacy

1.1.1 Definition of statical determinacy

In structural mechanics, **statical determinacy** refers to the condition where all internal forces and reactions in a structure can be determined solely from the equations of equilibrium. The equations of equilibrium are the basic mathematical relationships that ensure a structure remains at rest under applied loads. For a structure to be statically determinate, the number of unknown forces must equal the number of available equilibrium equations.

If the number of unknowns is greater than the number of equilibrium equations, the structure is said to be **indeterminate**, meaning additional methods such as compatibility equations or energy methods are required to solve it.

Fill in the blank: A structure is statically ________ when all forces can be found using only equilibrium equations.



1.1.2 Conditions for equilibrium in structures

For a structure to be in equilibrium, three fundamental conditions must be satisfied:

- The sum of all horizontal forces must be zero.
- The sum of all vertical forces must be zero.
- The sum of all moments about any point must be zero.

These conditions are expressed mathematically as: ΣFx=0,ΣFy=0,ΣM=0\Sigma F_x = 0, \Sigma F_y = 0, \Sigma M = 0.

These equations form the basis of structural analysis. If the number of unknown reactions or forces exceeds these three equations, the structure cannot be solved by equilibrium alone.

Fill in the blank: The three equilibrium conditions are based on forces in the horizontal direction, forces in the vertical direction, and ________.

Box 1.1 Summary of Equilbrium Conditions

For a rigid structure to remain to rest (rest (static equilbrium, it Newtisfy Nevton's First Law Motion. This requires that the net net force ant force and the moment on the body are zero. For two-dimensiood (planar) structure, these requirents into into three fundamental equations:

1. **Horizontal Equilbrium**
   The algerbic sum the horcal (x) direction must ts structure translatig sidways.
   $$\Sigma F_x = 0$$
2. **Vertical Equilbrium**
   The algeribic sum of all force components acting in the horizontal direction mo zero. This prevents the structure from translating sidways.
2. **Vertical Equibrium**
   The algeribic sum of all force compents acting the vertical (y) dienslatine up up or down.
   $$\Sigma F_y = 0$$
3. **Moment Equilbrium**
   The algebraic sne sum of the moments (turning forces) about any arbiarary O) nor off point (O) on or off stictre must the struıcture from rotating from rotating.
   $$\Sigma M_O = 0$$

1.1.3 Classification of structures: determinate and indeterminate

Structures are broadly classified into two categories:

- **Statically determinate structures**: These can be solved using only equilibrium equations. Examples include simply supported beams and basic trusses.
- **Statically indeterminate structures**: These require additional methods beyond equilibrium, such as compatibility conditions or energy methods, to determine forces. Examples include continuous beams and fixed-end frames.

This classification is important because it guides engineers in selecting the appropriate analysis method. Determinate structures are simpler to analyse, but indeterminate structures are often more economical and stable in practice.

Fill in the blank: A continuous beam with more than two supports is usually classified as statically ________.

Table 1.1 Comparison of Determinate and Indermentate Structures

Feature	Determinate Structures	Indiermimate Structures
Definition	Static equllibrium equations {$= F_x \; F_y = 0$, $F_x = M$,0) are suffficient to find to find all reactions & internal forces.	Static equlibrium equations are NOT sufficient to find all reactions & inffiorres. Additional (compatibilite of dispalcement) are required.
Analysis Method	Direct supplicated beams, Cantialive arches, equations.	Equilbilum + Compatibilite; Flexibility or Stiffsens fiffses methods (e. g., Moment Distribution, Slope-Deflection, Finite Element)
Examples	Simply supported Cantialver beams, Three-hinged arches, Simple trusses.	Contniuous beams, Fixed-end beams, Frames, Frames, Arches with two hinges or fixed supports, Propped cantiivers

Pilot questions 1.1

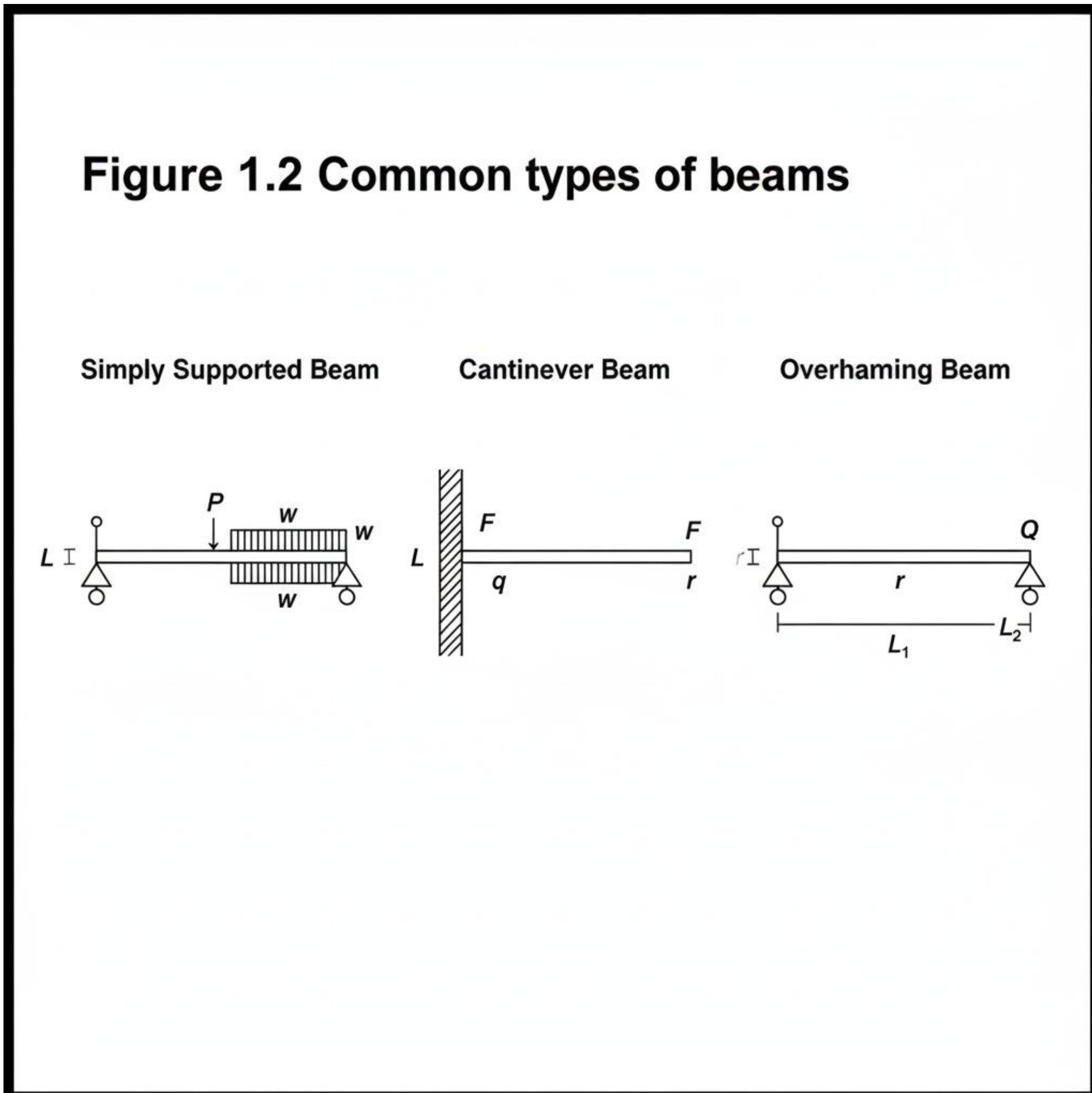
1. Define statical determinacy.
2. State the three fundamental conditions of equilibrium.
3. What happens when the number of unknowns exceeds the number of equilibrium equations?
4. Give one example of a statically determinate structure.
5. Give one example of a statically indeterminate structure.

1.2 Types Of Statically Determinate Structures

1.2.1 Beams

A **beam** is a structural member designed to resist loads applied perpendicular to its longitudinal axis. Beams are among the simplest statically determinate structures because their reactions can be calculated directly using equilibrium equations. Common examples include simply supported beams, cantilever beams, and overhanging beams.

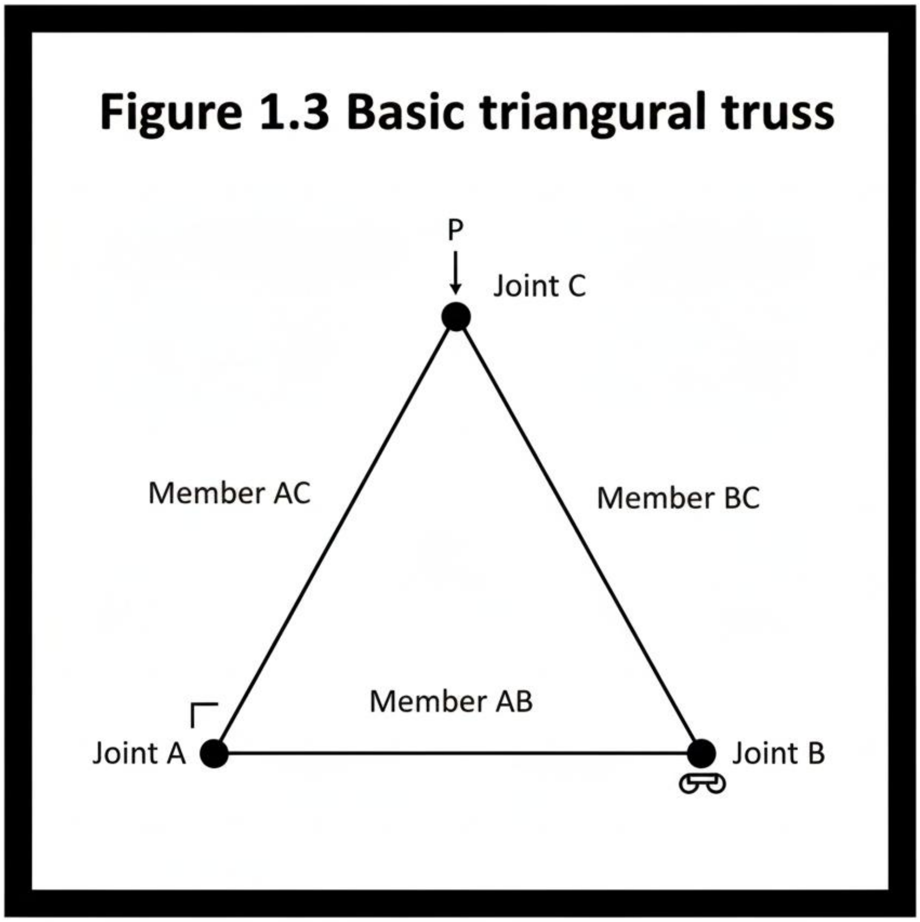
Fill in the blank: A beam is a structural member that resists loads applied ________ to its longitudinal axis.



1.2.2 Trusses

A **truss** is a structure composed of straight members connected at joints, usually arranged in triangular units. Trusses are widely used in bridges and roof systems because they efficiently transfer loads through axial forces in their members. In statically determinate trusses, the forces in each member can be found using equilibrium equations applied at the joints.

Fill in the blank: A truss is composed of straight members connected at ________, forming triangular units.

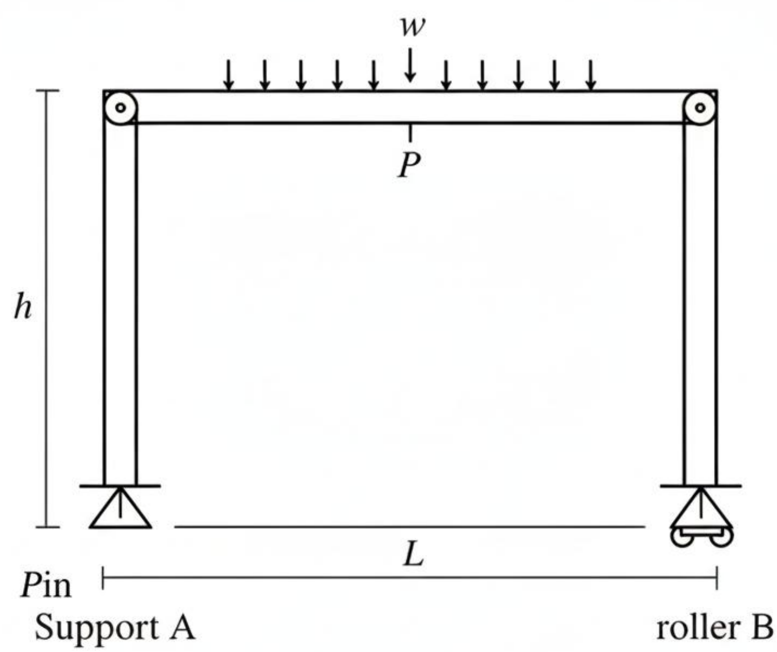


1.2.3 Frames

A **frame** is a structure consisting of beams and columns connected together, often forming rectangular or square units. Frames are common in building construction and can be either statically determinate or indeterminate depending on the number of supports and connections. In determinate frames, equilibrium equations are sufficient to calculate internal forces and reactions.

Fill in the blank: A frame is a structure consisting of beams and ________ connected together.

Figure 1.4 Simple rectangural frame



Pilot questions 1.2

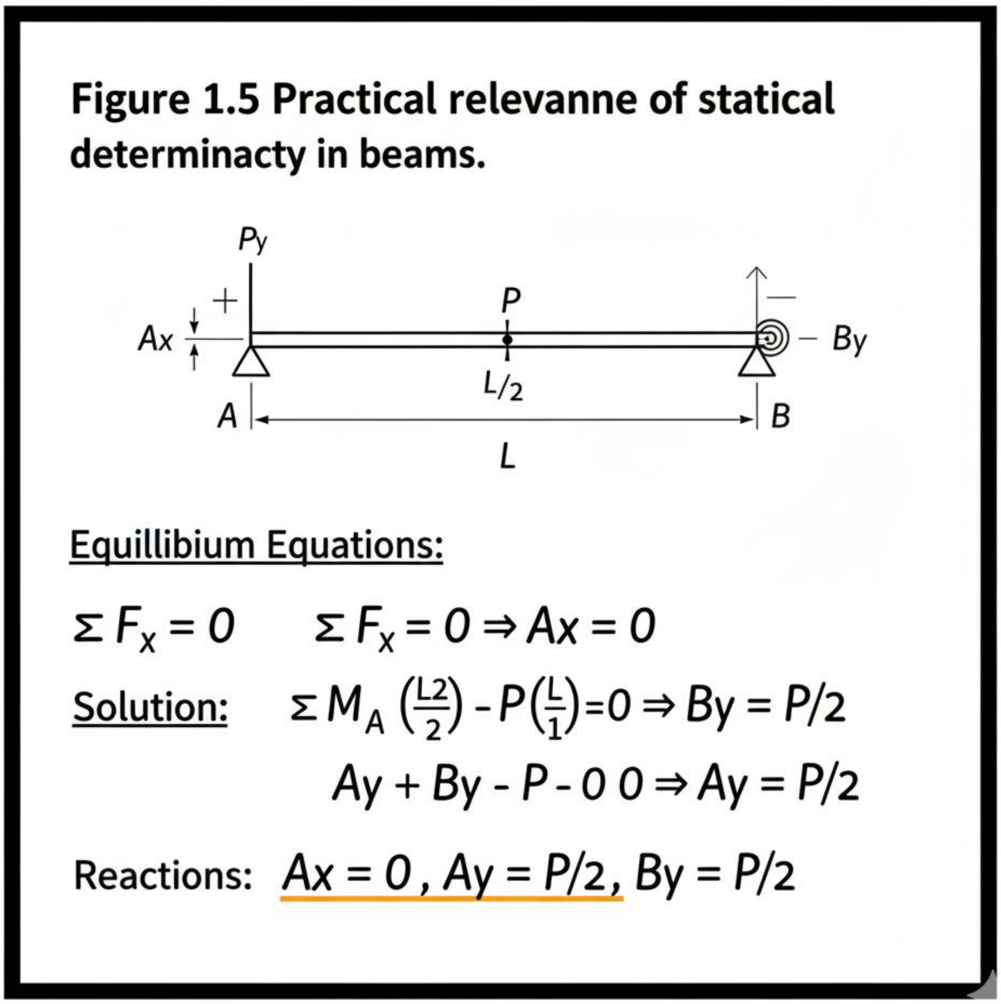
1. Define a beam and give one example of a statically determinate beam.
2. What geometric shape is commonly used in trusses to ensure stability?
3. State one practical application of trusses in engineering.
4. What structural members make up a frame?
5. Explain how a frame can be classified as determinate or indeterminate.

1.3 Importance And Applications Of Statical Determinacy

1.3.1 Practical relevance in structural analysis

The concept of **statical determinacy** is central to structural analysis because it allows engineers to calculate forces and reactions using only equilibrium equations. This makes determinate structures easier to analyse, especially in the early stages of design. For example, when designing a simply supported beam, the reactions at the supports can be determined directly without complex calculations.

Fill in the blank: Determinate structures can be analysed using only ________ equations.



1.3.2 Advantages and limitations of determinate structures

Advantages of determinate structures include simplicity of analysis, ease of teaching and learning, and clear understanding of load paths. They are often used in basic designs where quick calculations are required. **Limitations** include reduced redundancy, meaning failure of one member can lead to collapse, and less adaptability to complex loading conditions compared to indeterminate structures.

Fill in the blank: One limitation of determinate structures is reduced ________, which can lead to collapse if a member fails.

Box 1.2 Advantages and limitations of determate structures

Advantages

1. Easier to analyze using basic equilsblium equations.
2. Often more economial for short-span structures.
3. Simple construction a inspection.
4. Redudcy not present (clear load path).

Limitations

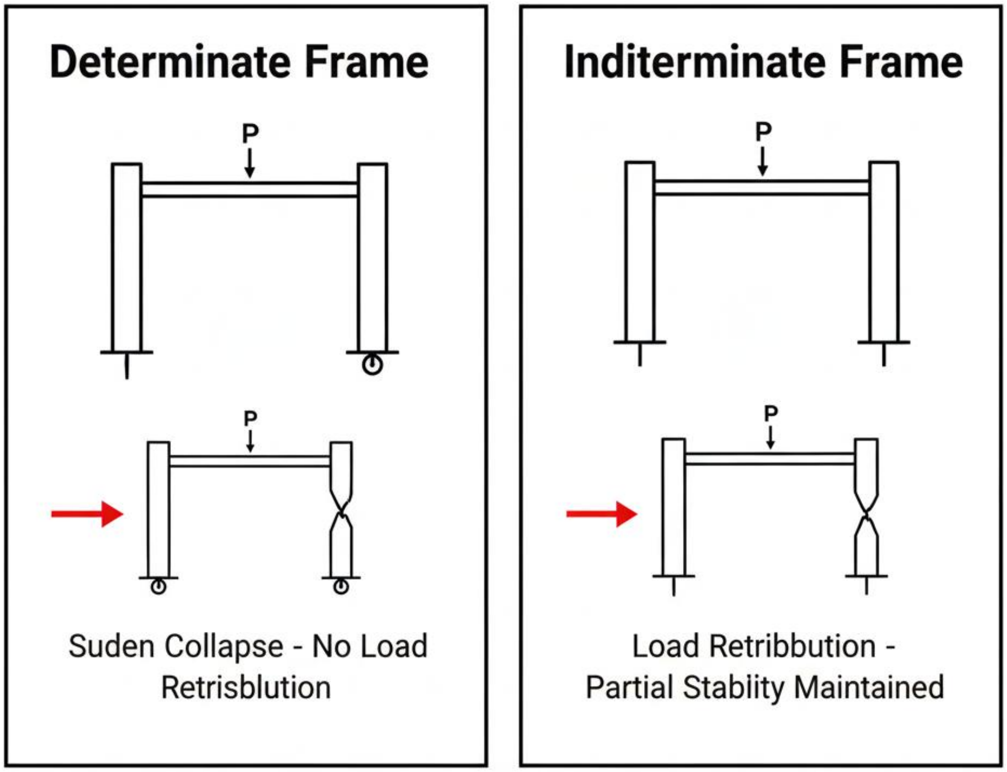
1. Lack of redurduncy (suden dden collapse if a member fails).
2. Less stiff, leading to larger defections.
3. Not suitable for long spans or complex structrex structures.
4. Sensitive to support settlements and tenperature changes.

1.3.3 Transition to indeterminate structures

While determinate structures are useful for learning and simple applications, most real-world structures are **indeterminate**. Indeterminate structures provide greater safety and economy because they can redistribute loads if one member fails. This makes them more reliable in practice, especially for bridges, multi-storey buildings, and continuous beams. Understanding statical determinacy prepares learners to appreciate why indeterminate structures are necessary and how they are analysed using advanced methods.

Fill in the blank: Indeterminate structures are often more ________ because they can redistribute loads when a member fails.

Figure 1.6 Comparison of Determinate and Indiemnate Frames



Pilot questions 1.3

1. Why are determinate structures easier to analyse?
2. State one advantage of determinate structures.
3. State one limitation of determinate structures.
4. Why are indeterminate structures more reliable in practice?
5. Give one example of a real-world indeterminate structure.

Series Summary

This Series has introduced you to the fundamental concept of statical determinacy, which is essential for analysing and classifying structures in structural mechanics. By working through the content, you have seen how equilibrium conditions form the basis of analysis and how structures can be categorised as determinate or indeterminate depending on whether equilibrium equations alone are sufficient.

We began by defining statical determinacy and examining the conditions for equilibrium, then moved on to explore the main types of statically determinate structures including beams, trusses, and frames. Finally, we considered the practical importance of statical determinacy, highlighting its advantages, limitations, and its role as a foundation for studying indeterminate structures. In line with the Series Learning Outcomes, you should now be able to define and classify structures, identify examples of determinate forms, explain their relevance in practice, and evaluate their strengths and weaknesses.

Glossary of Terms

- **Beam**: A structural member designed to resist loads applied perpendicular to its length, commonly used in bridges and buildings.
- **Equilibrium**: The condition in which all forces and moments acting on a structure balance out, keeping the structure at rest.
- **Frame**: A structure made up of beams and columns connected together, often forming rectangular or square units, widely used in building construction.
- **Indeterminate structure**: A structure in which the number of unknown forces exceeds the available equilibrium equations, requiring advanced methods such as compatibility or energy techniques for analysis.
- **Statical determinacy**: The property of a structure where all internal forces and reactions can be calculated using only the equations of equilibrium.
- **Statically determinate structure**: A structure that can be fully analysed using equilibrium equations alone, such as a simply supported beam or a basic truss.
- **Truss**: A structure composed of straight members connected at joints, usually arranged in triangular units, designed to carry loads efficiently through axial forces.

Concept Check Questions 1

Objective questions

1. A structure is statically determinate when: a) The number of unknowns is greater than equilibrium equations b) The number of unknowns equals equilibrium equations c) The number of equilibrium equations is greater than unknowns d) None of the above (Linked to SLO 1.1)
2. Which of the following is not a statically determinate structure? a) Simply supported beam b) Cantilever beam c) Continuous beam with three supports d) Basic triangular truss (Linked to SLO 1.2)
3. The three fundamental equilibrium conditions are based on: a) Horizontal forces, vertical forces, and moments b) Axial forces, shear forces, and bending moments c) Loads, reactions, and deflections d) None of the above (Linked to SLO 1.1)

Short-answer questions

4. Define statical determinacy in your own words. (Linked to SLO 1.1)
5. Give one example each of a determinate beam, truss, and frame. (Linked to SLO 1.3)
6. State one advantage and one limitation of determinate structures. (Linked to SLO 1.5)

Long-answer questions

7. Explain why equilibrium equations alone are sufficient for analysing determinate structures, and discuss what happens when the number of unknowns exceeds the number of equations. (Linked to SLO 1.1)

- **Basic response:** Equilibrium equations balance forces and moments. When the number of unknowns equals the equations, all forces can be solved. If unknowns exceed equations, the structure becomes indeterminate and requires additional methods.
- **Value-added response:** Outstanding learners may expand by discussing how compatibility conditions, deformation analysis, or energy methods are used to solve indeterminate structures, and why engineers often prefer indeterminate designs for safety and redundancy.

8. Analyse the importance of trusses in engineering practice, highlighting their structural efficiency and common applications. (Linked to SLO 1.3)

- **Basic response:** Trusses are efficient because they transfer loads through axial forces in members. They are commonly used in bridges and roofs.
- **Value-added response:** Outstanding learners may add that trusses minimise material usage, allow for large spans, and can be designed in various configurations (e.g., Pratt, Warren, Howe) to suit specific engineering needs.

9. Evaluate the advantages and limitations of determinate structures, and explain why indeterminate structures are often preferred in real-world applications. (Linked to SLO 1.5)

- **Basic response:** Determinate structures are simple to analyse but lack redundancy. Indeterminate structures are more reliable because they redistribute loads.
- **Value-added response:** Outstanding learners may elaborate by discussing cost efficiency, resilience under unexpected loads, and how modern computational tools make the analysis of indeterminate structures practical and widespread.

Answers to Concept Check Questions 1

Objective questions

1. b) The number of unknowns equals equilibrium equations
2. c) Continuous beam with three supports
3. a) Horizontal forces, vertical forces, and moments

Short-answer questions

4. Statical determinacy is the condition where all forces and reactions in a structure can be determined using only equilibrium equations.
5. Examples: Simply supported beam (beam), triangular truss (truss), simple rectangular frame (frame).
6. Advantage: Easy to analyse. Limitation: Reduced redundancy.

Long-answer questions

7.

- **Basic response:** Equilibrium equations are sufficient when unknowns equal equations. If unknowns exceed equations, the structure is indeterminate.
- **Value-added response:** Indeterminate structures require compatibility or energy methods, and engineers often prefer them for redundancy and safety.

8.

- **Basic response:** Trusses transfer loads through axial forces and are used in bridges and roofs.
- **Value-added response:** They minimise material use, allow large spans, and can be designed in different configurations for efficiency.

9.

- **Basic response:** Determinate structures are simple but lack redundancy. Indeterminate structures redistribute loads and are more reliable.
- **Value-added response:** They are cost-efficient, resilient under unexpected loads, and modern tools make their analysis practical.

Answers to Pilot Questions 1

Part 1.1 Concept of Statical Determinacy

- Statical determinacy is the condition where all internal forces and reactions can be determined using only equilibrium equations.
- The three fundamental conditions of equilibrium are: the sum of horizontal forces equals zero, the sum of vertical forces equals zero, and the sum of moments equals zero.
- When the number of unknowns exceeds the number of equilibrium equations, the structure is statically indeterminate.
- Example of a statically determinate structure: simply supported beam.
- Example of a statically indeterminate structure: continuous beam with more than two supports.

Part 1.2 Types of Statically Determinate Structures

- A beam is a structural member that resists loads applied perpendicular to its longitudinal axis; example: cantilever beam.
- The geometric shape commonly used in trusses to ensure stability is the triangle.
- Practical application of trusses: roof structures or bridges.
- Frames are made up of beams and columns connected together.
- A frame is classified as determinate if equilibrium equations are sufficient to solve for all unknowns, otherwise it is indeterminate.

Part 1.3 Importance and Applications of Statical Determinacy

- Determinate structures are easier to analyse because their forces can be found using only equilibrium equations.
- One advantage of determinate structures is simplicity of analysis.
- One limitation of determinate structures is reduced redundancy.
- Indeterminate structures are more reliable because they can redistribute loads when a member fails.
- Example of a real-world indeterminate structure: multi-storey building frame.

References and Attributions 1

This section compiles all references and illustration attributions used or suggested in **Series 1: Statical Determinacy of Structures: Fundamentals**. The referencing style follows APA guidelines for clarity and consistency.

General References

- British Geological Survey. (n.d.). *Structural mechanics and equilibrium principles*. Retrieved from https://www.bgs.ac.uk
- Open University. (n.d.). *Open educational resources on structural analysis*. Retrieved from https://www.open.edu
- United States Geological Survey. (n.d.). *Basic concepts in soil and structural mechanics*. Retrieved from https://www.usgs.gov

Illustrations

- *Figure 1.1 Simple beam showing statical determinacy* Attribution: Suggested OER source – USGS diagrams on equilibrium beams. AI-generated prompt: "Generate a diagram of a simply supported beam with applied load, showing equilibrium equations." Tool/platform: Microsoft Copilot.
- *Box 1.1 Summary of equilibrium conditions* Attribution: Suggested OER source – Open University teaching resources. AI-generated prompt: "Create a text box summarising equilibrium conditions for structures with equations." Tool/platform: Microsoft Copilot.
- *Table 1.1 Comparison of determinate and indeterminate structures* Attribution: Suggested OER source – British Geological Survey resources. AI-generated prompt:

"Create a comparison table of determinate vs indeterminate structures with examples and features." Tool/platform: Microsoft Copilot.

- *Figure 1.2 Common types of beams* Attribution: Suggested OER source – USGS diagrams of beam types. AI-generated prompt: "Generate a diagram showing simply supported, cantilever, and overhanging beams with loads." Tool/platform: Microsoft Copilot.
- *Figure 1.3 Basic triangular truss* Attribution: Suggested OER source – Open University OER diagrams. AI-generated prompt: "Generate a diagram of a triangular truss showing joints and members." Tool/platform: Microsoft Copilot.
- *Figure 1.4 Simple rectangular frame* Attribution: Suggested OER source – British Geological Survey resources. AI-generated prompt: "Generate a diagram of a rectangular frame showing beams, columns, supports, and loads." Tool/platform: Microsoft Copilot.
- *Figure 1.5 Practical relevance of statical determinacy in beams* Attribution: Suggested OER source – USGS diagrams on equilibrium reactions. AI-generated prompt: "Generate a diagram of a simply supported beam showing reactions calculated using equilibrium equations." Tool/platform: Microsoft Copilot.
- *Box 1.2 Advantages and limitations of determinate structures* Attribution: Suggested OER source – Open University resources. AI-generated prompt: "Create a text box listing advantages and limitations of determinate structures." Tool/platform: Microsoft Copilot.
- *Figure 1.6 Comparison of determinate and indeterminate frames* Attribution: Suggested OER source – British Geological Survey diagrams. AI-generated prompt: "Generate a diagram comparing determinate and indeterminate frames under load redistribution." Tool/platform: Microsoft Copilot.

Series 2:Analysis of Statically Determinate Trusses

- **Part 2.1 Fundamentals of Trusses**
  - 2.1.1 Definition and characteristics of trusses
  - 2.1.2 Assumptions in truss analysis
  - 2.1.3 Classification of trusses (simple, compound, complex)
- **Part 2.2 Methods of Truss Analysis**
  - 2.2.1 Method of joints
  - 2.2.2 Method of sections
  - 2.2.3 Graphical methods (Maxwell diagram, funicular polygon)

- **Part 2.3 Applications and Case Studies**
  - 2.3.1 Practical examples of truss structures (bridges, roofs, towers)
  - 2.3.2 Advantages and limitations of determinate trusses
  - 2.3.3 Case studies in engineering practice

Introduction

In the last Series, we explored the fundamentals of statical determinacy and established the conditions under which structures can be classified as determinate or indeterminate. Building on that foundation, we now turn our attention to trusses, which are among the most widely used structural systems in engineering. Trusses are essential in bridges, roofs, and towers, and their analysis provides a practical application of the principles of equilibrium and determinacy. Understanding how to analyse trusses equips you with the ability to evaluate forces in members and ensures that these structures can safely carry applied loads.

The aim of this Series is to introduce you to the analysis of determinate trusses and to equip you with the skills needed to apply different methods of analysis, interpret results, and appreciate their practical applications in engineering design.

We shall commence this Series by examining the fundamentals of trusses, including their definition, assumptions, and classification. Next, we will move on to the methods of truss analysis, focusing on the method of joints, the method of sections, and graphical approaches. Finally, we will wrap up by considering applications and case studies, where you will see how trusses are used in real-world structures, their advantages, and their limitations. This sequential flow will help you connect theory with practice and prepare you for more advanced structural analysis in later Series.

Series Learning Outcomes

Upon completing this Series, you should be able to:

- **SLO 2.1** define the fundamental characteristics of trusses and state the assumptions used in their analysis,
- **SLO 2.2** classify trusses into simple, compound, and complex forms,
- **SLO 2.3** demonstrate the method of joints in calculating member forces of determinate trusses,
- **SLO 2.4** apply the method of sections to determine forces in selected truss members,
- **SLO 2.5** illustrate graphical methods such as the Maxwell diagram and funicular polygon in truss analysis, and

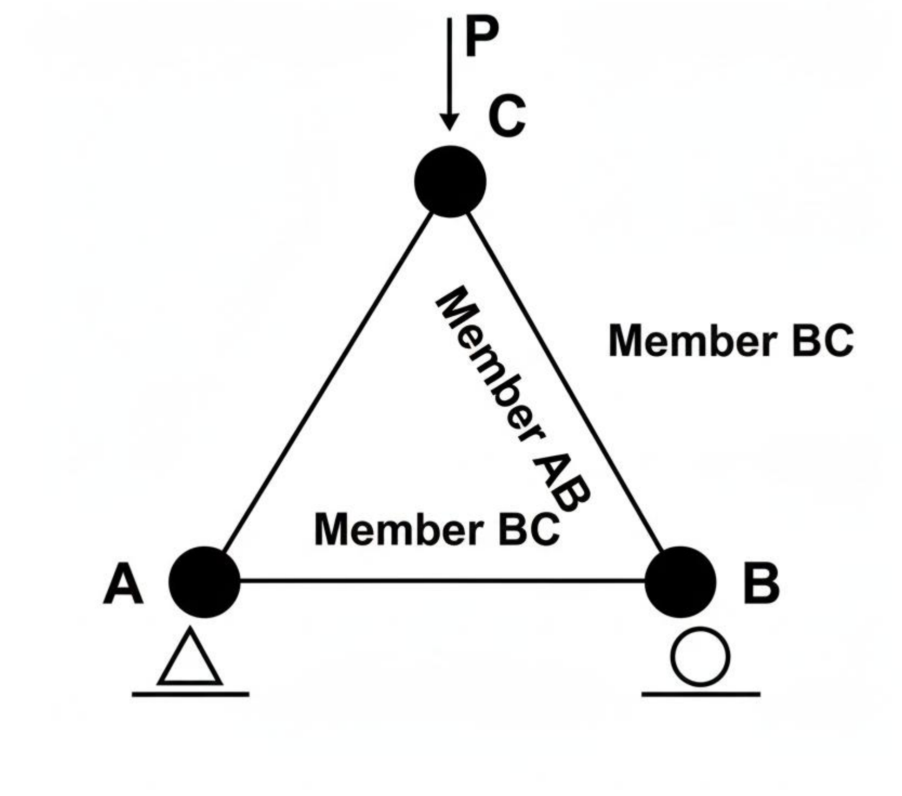
- **SLO 2.6** evaluate practical applications of determinate trusses by analysing case studies and identifying their advantages and limitations.

2.1 Fundamentals of Trusses

2.1.1 Definition and characteristics of trusses

A **truss** is a structural framework composed of straight members connected at their ends by joints, usually arranged in triangular units. The triangular configuration ensures stability and allows loads to be transferred efficiently through axial forces in the members. In most cases, trusses are assumed to have **pin joints**, meaning the connections allow rotation but not translation. This assumption simplifies analysis by ensuring that members carry only axial forces, either tension or compression.

Figure 2.1 Simple triangulal truss with pin joints



Fill in the blank: A truss is a framework made up of straight members connected at ________.

2.1.2 Assumptions in truss analysis

When analysing trusses, engineers make certain assumptions to simplify calculations. The most common assumptions are:

- All joints are **pin joints**, which means members are free to rotate but not translate.
- Loads are applied only at the joints, not along the members.
- Members are perfectly straight and weightless, carrying only axial forces.
- The truss is statically determinate, meaning equilibrium equations alone are sufficient to solve for member forces.

These assumptions may not always hold true in practice, but they provide a reliable basis for analysis in most engineering applications.

Box 2.1 Assumptions in Truss Analysis

- 1. Members are connected by frichholess pin joints, allowing free rotation.
- 2. External loads and reactions are applied only ont at the joints (no mid-member loads).
- 3. Members are straight and only cry axial forces (either teither tension or compression), making them two-force members.
- 4. The weight of the members is negligible compared to the applied loads, or is distributed to the joints.

Fill in the blank: In truss analysis, it is assumed that loads act only at the ________.

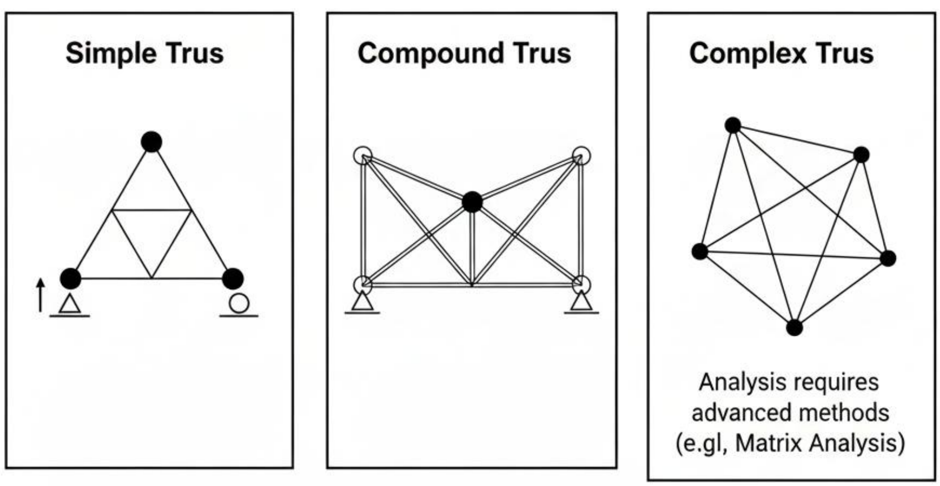
2.1.3 Classification of trusses

Trusses can be classified into three main types:

- **Simple trusses**, which are built from a basic triangular unit and extended by adding more triangles.
- **Compound trusses**, formed by connecting two or more simple trusses together.
- **Complex trusses**, which do not follow the simple triangular pattern and may require advanced methods for analysis.

Each type of truss has its own applications. Simple trusses are common in small roof structures, compound trusses are used in bridges, and complex trusses are often found in large towers or industrial frameworks.

Figure 2.2 Classification of trusses: simple, compound, and Complex



Fill in the blank: A truss formed by connecting two or more simple trusses together is called a ________ truss.

Pilot questions 2.1

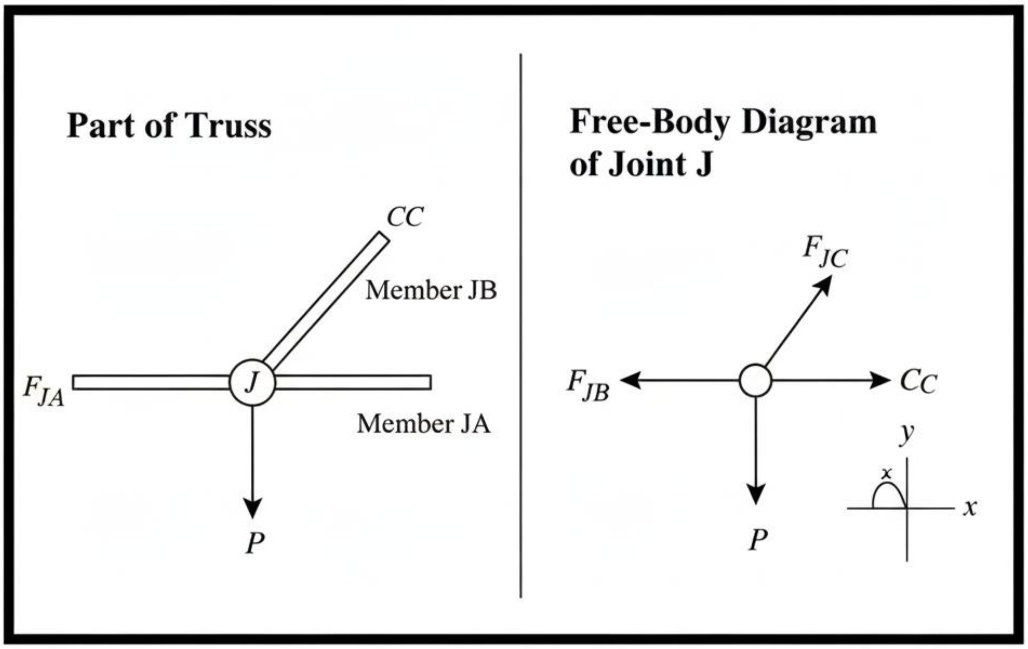
1. Define a truss and state its main structural characteristic.
2. List two assumptions commonly made in truss analysis.
3. What type of truss is formed by connecting two or more simple trusses?
4. Which type of truss is most commonly used in roof structures?
5. Why are triangular units important in truss design?

2.2 Methods of Truss Analysis

2.2.1 Method of joints

The **method of joints** is a systematic approach used to determine the forces in truss members by applying the equations of equilibrium at each joint. Since each joint is assumed to be a pin connection, the sum of forces in both the horizontal and vertical directions must equal zero. By isolating one joint at a time and solving for the unknown member forces, the entire truss can be analysed. This method is particularly useful when you need to determine the forces in many members.

Figure 2.3 Method of joints applied to a truss joint



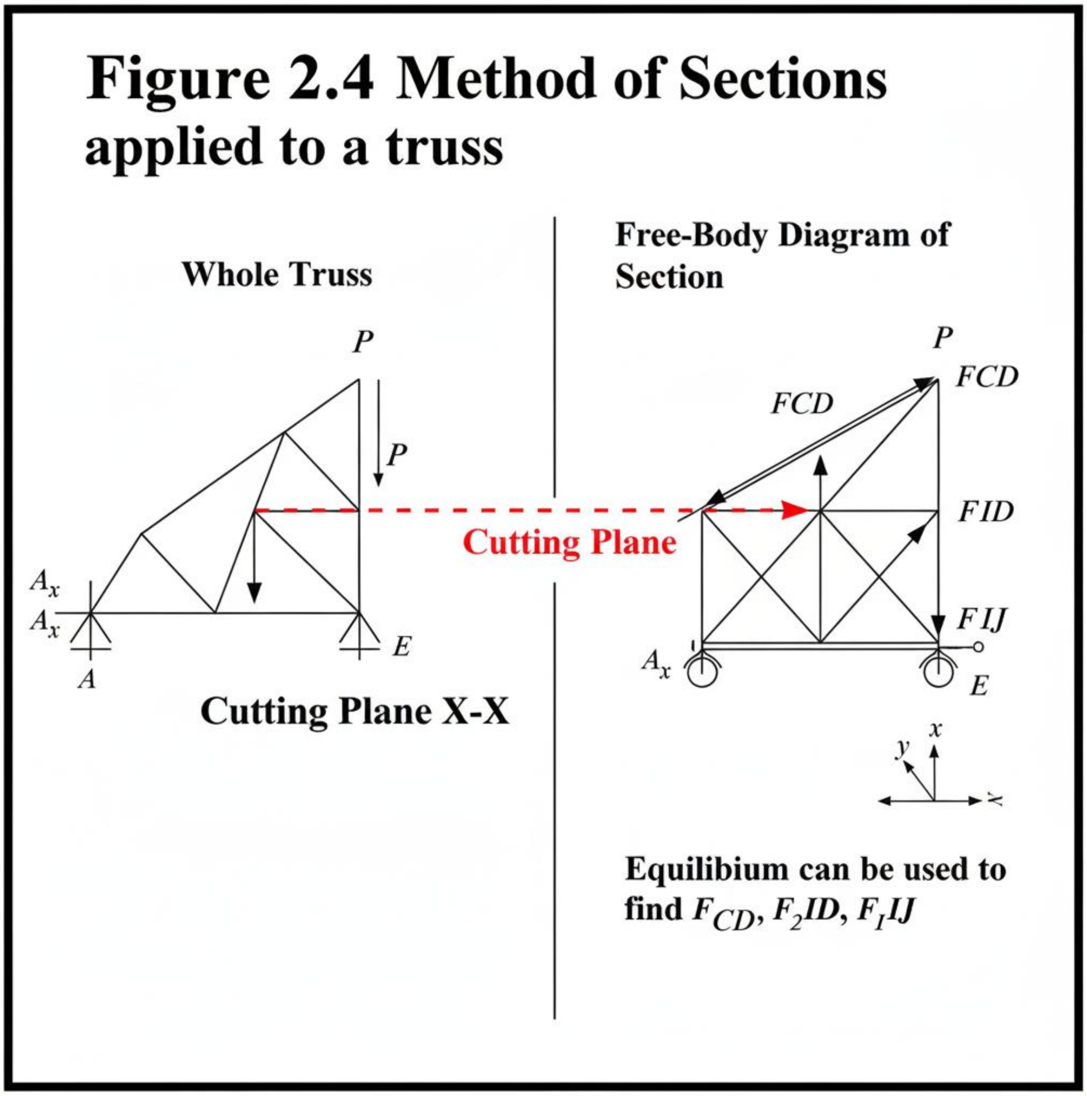
$$\Sigma F_x = 0 \Rightarrow F_{JB} - F_{JB} - F_{JA} + F_{JC} \cdot C_{JC} * \cos() = 0$$

$$\Sigma F_y = 0 \Rightarrow \quad = F_{CC} * \sin * \cos\theta - P = 0$$

Fill in the blank: In the method of joints, the sum of forces at each ________ must equal zero.

2.2.2 Method of sections

The **method of sections** involves cutting through the truss to expose a section and then applying equilibrium equations to solve for the forces in specific members. This method is efficient when you only need to find the forces in a few selected members rather than analysing the entire truss. By making a cut that passes through no more than three unknown members, the equations of equilibrium can be applied directly to determine their forces.

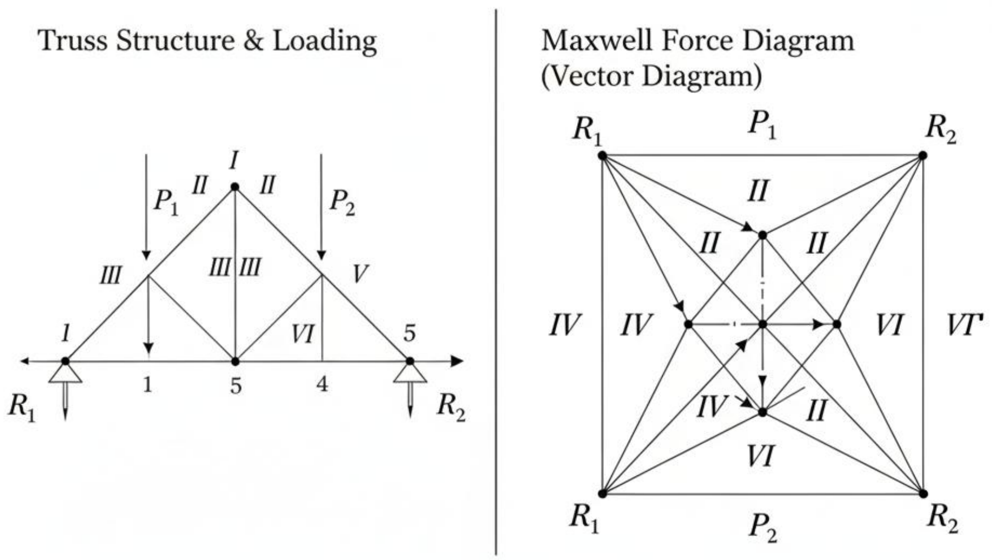


Fill in the blank: In the method of sections, the cut should pass through no more than ________ unknown members.

2.2.3 Graphical methods

Graphical methods such as the Maxwell diagram and the funicular polygon provide a visual approach to truss analysis. These methods involve constructing scaled diagrams to represent forces and geometry, allowing member forces to be determined graphically. While less common today due to computational tools, graphical methods are valuable for visualising force distribution and for manual checks of analytical results.

Figure 2.5 Maxwell diagram for truss analysis



Fill in the blank: Graphical methods such as the Maxwell diagram use ________ diagrams to determine member forces.

Pilot questions 2.2

1. What is the main principle used in the method of joints?
2. How many unknown members should a section cut pass through in the method of sections?
3. Which method is more efficient when only a few member forces are required?
4. Name one graphical method used in truss analysis.
5. Why are graphical methods still useful despite modern computational tools?

2.3 Applications and Case Studies

2.3.1 Practical examples of truss structures

Trusses are widely used in engineering because they provide strength and stability while using relatively little material. Common applications include **bridges**, where trusses distribute loads efficiently across spans, **roof structures**, which rely on trusses to support coverings and resist wind loads, and **towers**, where trusses provide rigidity against lateral forces. These examples highlight how trusses combine simplicity with effectiveness in real-world projects.

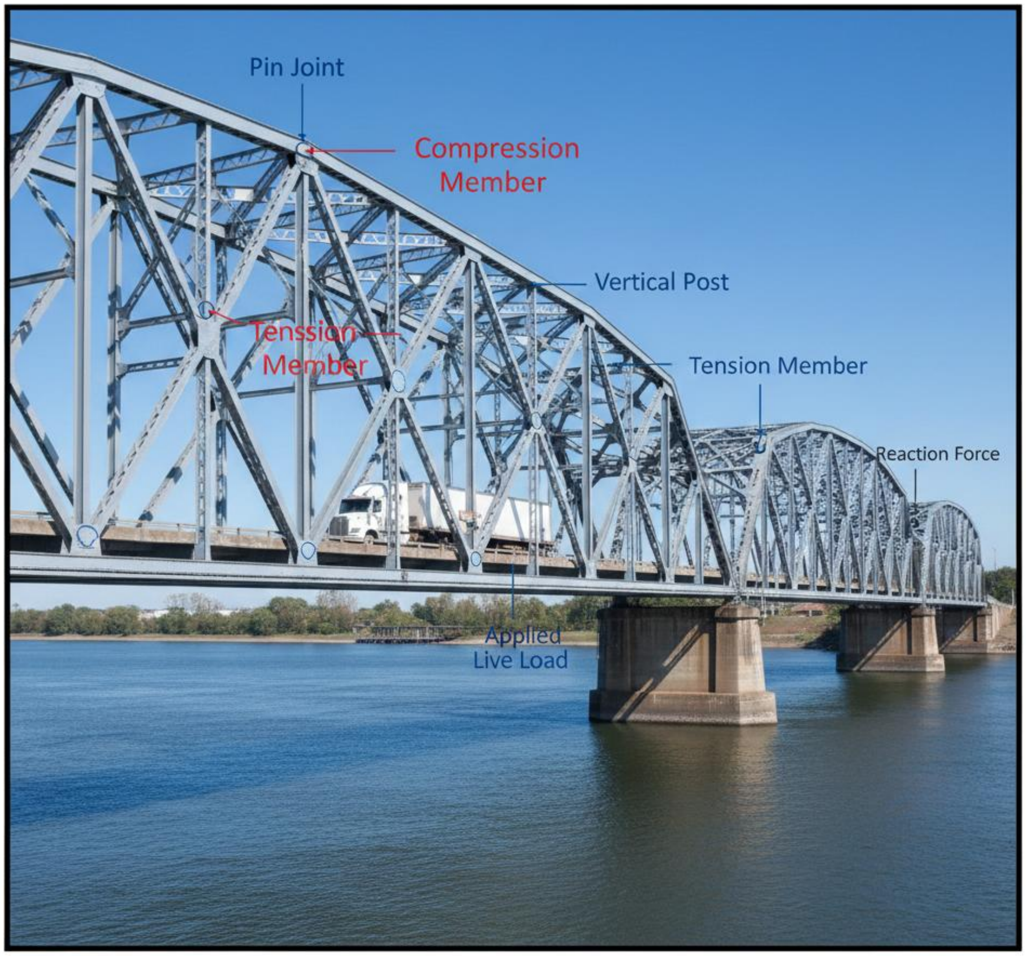


Plate 2.1 Truss bridge showing load distribution.

Fill in the blank: Roof structures often rely on ________ to provide support and resist wind loads.

2.3.2 Advantages and limitations of determinate trusses

Determinate trusses offer several advantages. They are relatively simple to analyse using equilibrium equations, and their member forces can be calculated without complex methods. They also provide efficient load distribution and are cost-effective in many applications. However, determinate trusses have limitations. They may lack redundancy, meaning that if one member fails, the entire structure could collapse. They are also less adaptable to complex loading conditions compared to indeterminate trusses.

Box 2.2 Advantages and limitations of determate trusses

Advantages

1. Simple to analyze using statics (equations of equilbilium).
2. Often lighter, requring less material than indenbuate trusses.
3. Easier of fablricate and erect due due simpler geometry.
4. Internal forces are not affected by small support setlements or temperature changes.
5. Clear load path, making it easier it abwie vissalize o visalize force distribution.

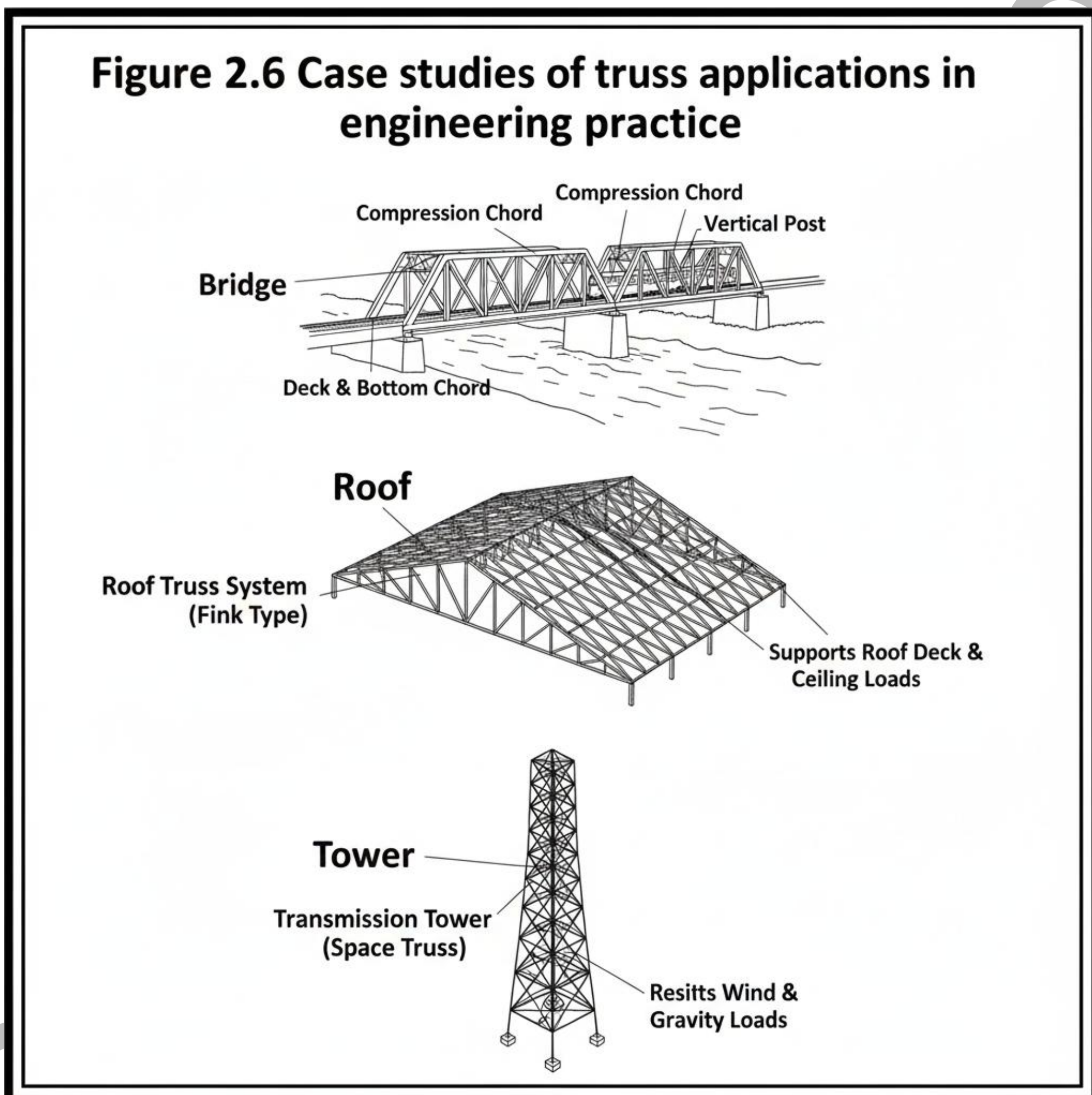
Limitations

1. Lack of redundvaicy: failure one member can lead of catarasaptic collapse.
2. Less stiff, resulting byt large-load.
2. Not suitable for larger larger deffections under loads.
4. Not suitable for very spans or heavy, concnerated loads.
4. Sensitive to fabrication errors (cannot absorb geometric imperfections as well).

Fill in the blank: A limitation of determinate trusses is that they may lack ________, making them vulnerable to collapse if a member fails.

2.3.3 Case studies in engineering practice

Case studies provide valuable insights into how trusses are applied in practice. For example, truss bridges in Nigeria have been used to span rivers and provide reliable transport routes. Roof trusses in public buildings such as schools and stadiums demonstrate their versatility in covering large spaces. Towers and transmission line supports also rely on truss frameworks to withstand environmental forces. These examples show how determinate trusses are integrated into everyday infrastructure.



Fill in the blank: Transmission line supports often use ________ frameworks to withstand environmental forces.

Pilot questions 2.3

1. Name two common applications of trusses in engineering.
2. What is one major advantage of determinate trusses?

3. What is one limitation of determinate trusses?
4. Give an example of a truss application in Nigeria.
5. Why are case studies important in understanding truss applications?

Series Summary

This Series has focused on the analysis of determinate trusses, an essential aspect of structural mechanics that builds directly on the principles of statical determinacy. Its purpose has been to help you recognise the role of trusses in engineering practice and to equip you with the analytical tools needed to evaluate their behaviour under load. By working through the content, you have seen how trusses form the backbone of bridges, roofs, and towers, and why their study is central to safe and efficient design.

We began by exploring the fundamentals of trusses, including their definition, assumptions, and classification. Then we moved on to the methods of truss analysis, where you applied the method of joints, the method of sections, and graphical approaches such as the Maxwell diagram. Finally, we examined applications and case studies, highlighting both the advantages and limitations of determinate trusses in practice. In line with the Series Learning Outcomes, you should now be able to define and classify trusses, demonstrate and apply analytical methods to calculate member forces, illustrate graphical techniques, and evaluate practical applications through case studies. This provides you with a solid foundation for more advanced structural analysis in subsequent Series.

Glossary of Terms

- **Axial force**: A force that acts along the length of a structural member, either stretching it in tension or compressing it.
- **Case study**: A detailed example used to illustrate how trusses are applied in real engineering projects such as bridges, roofs, or towers.
- **Complex truss**: A truss that does not follow the simple triangular pattern and often requires advanced methods for analysis.
- **Compound truss**: A truss formed by connecting two or more simple trusses together to create a larger framework.
- **Determinate truss**: A truss in which the forces in all members can be found using only the equations of equilibrium.
- **Funicular polygon**: A graphical method used to determine forces in truss members by constructing a polygon that represents the equilibrium of forces.
- **Graphical methods**: Techniques such as the Maxwell diagram and funicular polygon that use scaled drawings to determine member forces visually.
- **Joint**: The point where two or more truss members meet, usually assumed to be a pin connection that allows rotation but not translation.
- **Maxwell diagram**: A graphical method of truss analysis that uses force polygons to determine the magnitude and direction of member forces.

- **Method of joints**: A technique for analysing trusses by applying equilibrium equations at each joint to calculate member forces.
- **Method of sections**: A technique for analysing trusses by cutting through the structure and applying equilibrium equations to determine forces in selected members.
- **Pin joint**: A type of connection in a truss that allows rotation but prevents translation, simplifying analysis by ensuring members carry only axial forces.
- **Simple truss**: A basic truss built from a triangular unit and extended by adding more triangles, commonly used in small structures such as roofs.
- **Truss**: A structural framework made of straight members connected at joints, arranged in triangular units to provide stability and efficient load transfer.

Concept Check Questions 2

Objective questions

1. Which assumption is made about truss joints in analysis? (Linked to SLO 2.1)
2. A truss formed by connecting two or more simple trusses is called a ________. (Linked to SLO 2.2)
3. In the method of sections, the cut should pass through no more than how many unknown members? (Linked to SLO 2.4)
4. Which graphical method uses force polygons to determine member forces? (Linked to SLO 2.5)
5. Which type of truss is most commonly used in roof structures? (Linked to SLO 2.2)

Short-answer questions

6. Define a truss and explain its main structural characteristic. (Linked to SLO 2.1)
7. State two advantages of determinate trusses. (Linked to SLO 2.6)
8. Describe the main difference between the method of joints and the method of sections. (Linked to SLO 2.3 and SLO 2.4)
9. Give one practical example of truss application in Nigeria. (Linked to SLO 2.6)
10. Explain why triangular units are important in truss design. (Linked to SLO 2.1)

Long-answer questions

11. Analyse the steps involved in applying the method of joints to a simple truss. (Linked to SLO 2.3)
12. Evaluate the advantages and limitations of determinate trusses, using case studies to support your answer. (Linked to SLO 2.6)

Answers to Concept Check Questions 2

Objective questions

1. Pin joints.

2. Compound truss.
3. Three.
4. Maxwell diagram.
5. Simple truss.

Short-answer questions

6. A truss is a structural framework made of straight members connected at joints, usually arranged in triangular units. Its main characteristic is that members carry only axial forces, either tension or compression.
7. They are simple to analyse using equilibrium equations and provide efficient load distribution.
8. The method of joints analyses forces at each joint systematically, while the method of sections cuts through the truss to solve for selected member forces directly.
9. Truss bridges spanning rivers in Nigeria.
10. Triangular units provide stability and prevent deformation, ensuring the truss remains rigid under load.

Long-answer questions

11.

- **Basic response:** The method of joints involves isolating one joint at a time, applying equilibrium equations in the horizontal and vertical directions, and solving for unknown member forces. This process is repeated until all member forces are determined.
- **Value-added response:** In practice, engineers often begin with joints where only two unknowns exist, making calculations easier. Modern computational tools can automate this process, but manual analysis remains important for verifying results and understanding force distribution.

12.

- **Basic response:** Determinate trusses are easy to analyse and cost-effective, but they lack redundancy and may collapse if a member fails.
- **Value-added response:** Case studies show that while determinate trusses are widely used in bridges and roofs, indeterminate trusses are often preferred in modern infrastructure because they provide redundancy and can better handle complex loading conditions. For example, stadium roofs and large towers often use indeterminate designs to ensure safety and resilience.

Answers to Pilot Questions 2

Part 2.1 Fundamentals of Trusses

- A framework made up of straight members connected at joints.
- At the joints.

- Compound truss.
- Simple truss.
- Triangular units provide stability and rigidity.

Part 2.2 Methods of Truss Analysis

- The sum of forces at each joint must equal zero.
- Three unknown members.
- Method of sections.
- Maxwell diagram.
- They provide visualisation and manual checks of analytical results.

Part 2.3 Applications and Case Studies

- Bridges and roofs.
- Simple to analyse and efficient load distribution.
- Lack of redundancy.
- Truss bridges in Nigeria.
- They show practical applications and reinforce theoretical knowledge.

References and Attributions 2

This section compiles all references and illustration attributions used or suggested in **Series 2: Analysis of Determinate Trusses**. The referencing style follows APA guidelines for clarity and consistency.

General References

- British Geological Survey. (n.d.). *Structural mechanics and truss behaviour*. Retrieved from https://www.bgs.ac.uk
- Open University. (n.d.). *Open educational resources on structural analysis*. Retrieved from https://www.open.edu
- United States Geological Survey. (n.d.). *Engineering applications of trusses*. Retrieved from https://www.usgs.gov

Illustrations

- *Figure 2.1 Simple triangular truss with pin joints* Attribution: Suggested OER source – USGS diagrams on truss structures. AI-generated prompt: “Generate a labelled diagram of a simple triangular truss showing joints and members” using Microsoft Copilot.
- *Box 2.1 Assumptions in truss analysis* Attribution: Suggested OER source – Open University resources on truss analysis. AI-generated prompt: “Create a text box summarising assumptions in truss analysis: pin joints, joint loads, axial forces” using Microsoft Copilot.

- *Figure 2.2 Classification of trusses: simple, compound, and complex* Attribution: Suggested OER source – British Geological Survey diagrams. AI-generated prompt: "Generate a diagram comparing simple, compound, and complex trusses with labels" using Microsoft Copilot.
- *Figure 2.3 Method of joints applied to a truss joint* Attribution: Suggested OER source – USGS diagrams on truss analysis. AI-generated prompt: "Generate a labelled diagram showing method of joints applied to a truss joint with forces" using Microsoft Copilot.
- *Figure 2.4 Method of sections applied to a truss* Attribution: Suggested OER source – Open University resources. AI-generated prompt: "Generate a diagram showing method of sections applied to a truss with cut line through members" using Microsoft Copilot.
- *Figure 2.5 Maxwell diagram for truss analysis* Attribution: Suggested OER source – British Geological Survey diagrams. AI-generated prompt: "Generate a diagram of Maxwell graphical method for truss analysis with labelled forces" using Microsoft Copilot.
- *Plate 2.1 Truss bridge showing load distribution* Attribution: Suggested OER source – USGS photographs of truss bridges. AI-generated prompt: "Generate an image of a steel truss bridge with labelled members" using Microsoft Copilot.
- *Box 2.2 Advantages and limitations of determinate trusses* Attribution: Suggested OER source – Open University resources. AI-generated prompt: "Create a text box summarising advantages and limitations of determinate trusses" using Microsoft Copilot.
- *Figure 2.6 Case studies of truss applications in engineering practice* Attribution: Suggested OER source – British Geological Survey diagrams. AI-generated prompt: "Generate a diagram showing truss applications in bridges, roofs, and towers with labels" using Microsoft Copilot.

Open Educational Resources (OERs)

- Open University OER materials on truss analysis, assumptions, and applications.
- USGS OER resources on truss structures and engineering case studies.
- British Geological Survey OER resources on graphical methods and truss classifications.

Series 3: Shear Forces, Bending Moments, Slopes and Deflections in Beams and Portal Frames

- **Part 3.1 Shear Forces and Bending Moments in Beams**
  - 3.1.1 Definition of shear force and bending moment
  - 3.1.2 Shear force and bending moment diagrams
  - 3.1.3 Common loading cases (point loads, distributed loads, couples)
- **Part 3.2 Slopes and Deflections in Beams**

Introduction

When engineers design bridges, buildings, or even simple roof structures, they must be able to predict how these elements will respond to loads. Beams and portal frames are among the most common structural members, and their behaviour under shear forces, bending moments, and deflections is central to safe and efficient design. Understanding these responses allows us to anticipate how structures will carry loads, resist deformation, and maintain stability. This Series builds directly on the foundation laid in the previous Series on trusses, extending your knowledge to more complex structural elements.

The aim of this Series is to equip you with the skills to analyse beams and portal frames by determining shear forces, bending moments, slopes, and deflections. You will also learn how these analyses contribute to practical design decisions in structural engineering.

We shall commence this Series by examining shear forces and bending moments in beams, including how to construct diagrams and interpret common loading cases. Next, we will move on to slopes and deflections in beams, exploring the mathematical relationships and methods used to calculate them. Finally, we will round off the Series with the analysis of portal frames, considering their structural behaviour, internal forces, and deflections. This sequential approach will ensure you gain a comprehensive understanding of how beams and frames respond to loads in real-world applications.

Series Learning Outcomes

Upon completing this Series, you should be able to:

- **SLO 3.1** define shear force and bending moment in beams,
- **SLO 3.2** construct shear force and bending moment diagrams for beams under different loading conditions,
- **SLO 3.3** explain the relationship between bending moment and deflection in beams,
- **SLO 3.4** apply analytical methods such as double integration and Macaulay's method to calculate slopes and deflections in beams,
- **SLO 3.5** analyse the structural behaviour of portal frames under applied loads,

- **SLO 3.6** determine shear forces, bending moments, and deflections in portal frames.

3.1 Shear Forces And Bending Moments In Beams

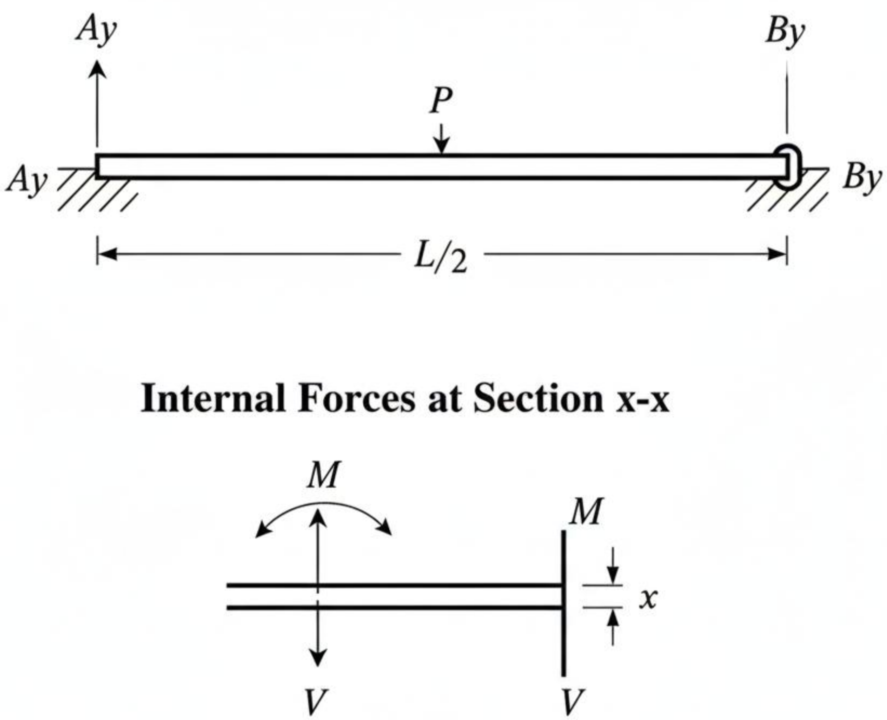
3.1.1 Definition of shear force and bending moment

When a beam is subjected to external loads, internal forces develop within it to resist those loads. **Shear force** is the internal force that acts parallel to the cross-section of the beam, tending to slide one part of the beam relative to the other. **Bending moment** is the internal moment that causes the beam to bend, resulting from the tendency of loads to rotate sections of the beam about a point.

These two quantities are fundamental in structural analysis because they help engineers determine whether a beam can safely carry the applied loads. If either the shear force or bending moment exceeds the material's capacity, the beam may fail.

Fill in the blank: The internal force that tends to slide one part of a beam relative to another is called ________.

Figure 3.1 Shear force and bending moment in a simply supported beam



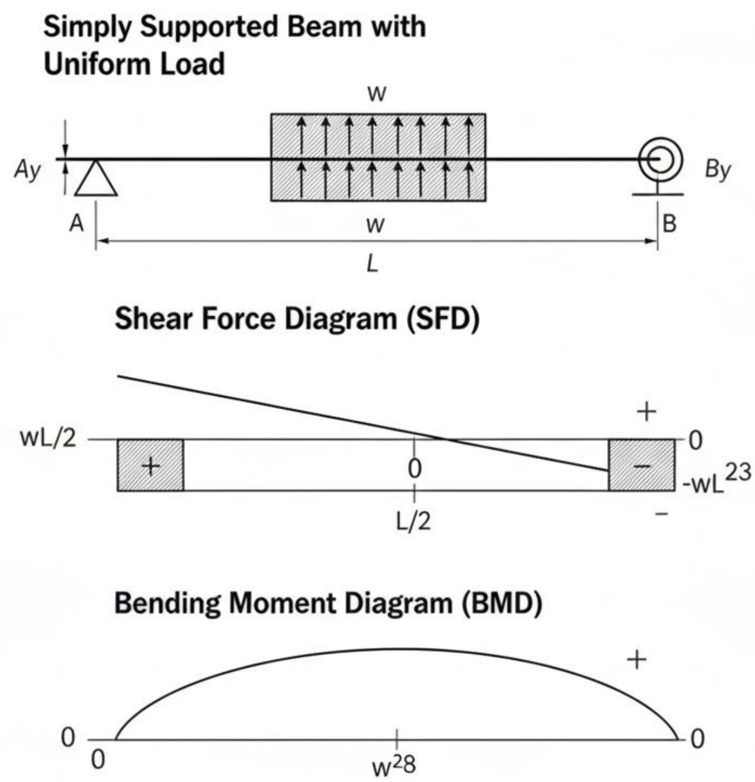
3.1.2 Shear force and bending moment diagrams

To visualise how shear forces and bending moments vary along the length of a beam, engineers use **shear force diagrams (SFDs)** and **bending moment diagrams (BMDs)**. These diagrams plot the magnitude of shear force and bending moment against the beam's span.

Constructing these diagrams involves calculating reactions at supports, then moving along the beam to determine internal forces at key points. The diagrams provide a clear picture of where maximum shear and bending occur, which is critical for design.

Fill in the blank: A diagram that shows how bending moment varies along the span of a beam is called a ________.

Figure 3.2 Shear force and bending moment diagrams for a unifomly loaded beam



3.1.3 Common loading cases

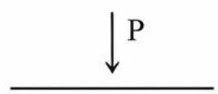
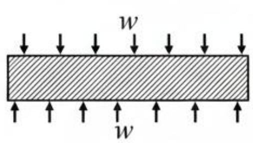
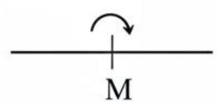
Beams are subjected to different types of loads in practice. The most common are:

- **Point loads**: Concentrated forces acting at a single point.
- **Uniformly distributed loads (UDLs)**: Loads spread evenly along the length of the beam.
- **Couples**: Pure moments applied to the beam without associated forces.

Each loading case produces a distinct pattern of shear force and bending moment. For example, a point load causes a sudden jump in the shear force diagram, while a UDL produces a linear variation.

Fill in the blank: A load that is spread evenly along the length of a beam is called a ________.

Box 3.1 Common loading cases for beams

Loading Type	Description	Effects on SFD & BMD
Point Load (P)		SFD: Vertical jump equal to P. BMD: Change in slope at the load.
Concentrated force at a single point		SFD: Linear change fore = -w).
Uniformly Distributed **Load** (w) 	Constant force per unit length 	SFD: Linear change in force (slope = -w). Parabic curve (concave down). SFD: No change.
Couple (M) applied at a point	Pure moment applied at a point	BMD: Vertical jump equal to M.

Pilot questions 3.1

1. Define shear force in a beam.
2. What is a bending moment diagram used for?
3. Identify one loading case that produces a linear variation in shear force.
4. Which type of load causes a sudden jump in the shear force diagram?
5. State the difference between point loads and uniformly distributed load

3.2 Slopes And Deflections In Beams

3.2.1 Relationship between bending moment and deflection

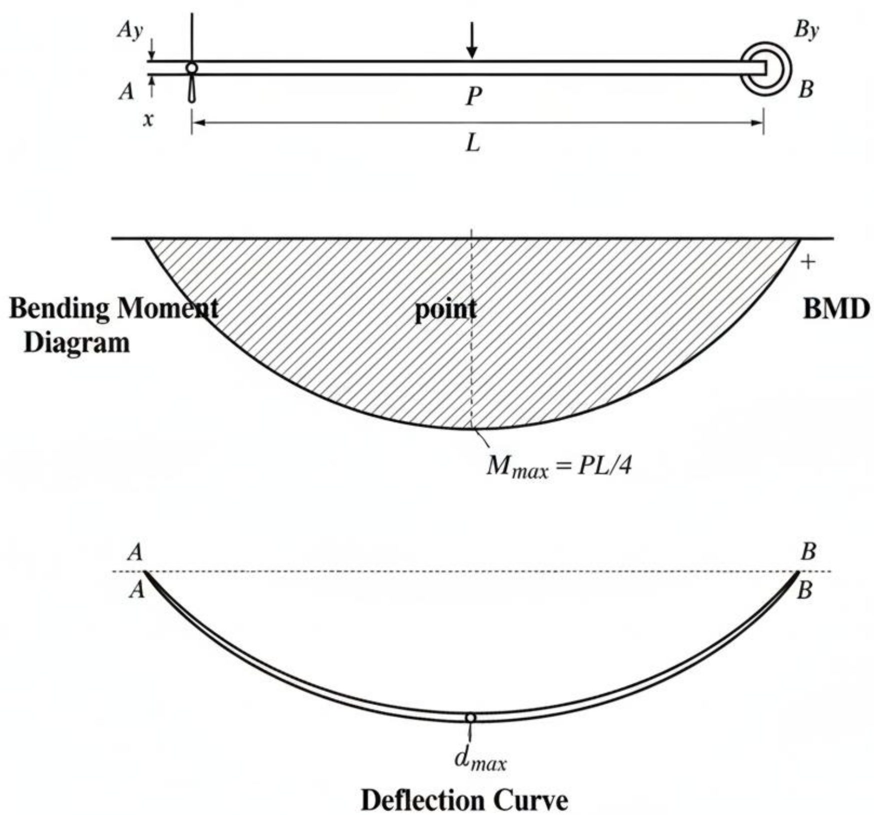
When a beam bends under load, the curvature of the beam is directly related to the **bending moment** at each point along its length. The mathematical relationship is expressed as:

d2ydx2=MEI\frac{d^2y}{dx^2} = \frac{M}{EI}

where yy is the deflection, MM is the bending moment, EE is the modulus of elasticity of the material, and II is the moment of inertia of the beam's cross-section. This equation shows that deflection depends not only on the applied loads but also on the material and geometry of the beam.

Fill in the blank: The deflection of a beam depends on the bending moment, modulus of elasticity, and ________.

Figure 3.3 Relationship between bending moment and deflection in a beam



3.2.2 Methods of calculating deflection

There are several methods used to calculate slopes and deflections in beams. The most common are:

- **Double integration method**: This involves integrating the bending moment equation twice to obtain slope and deflection.
- **Macaulay's method**: A simplified approach that uses step functions to handle discontinuous loading conditions.
- **Moment-area method**: A graphical technique based on the area under the bending moment diagram.

Each method has its advantages. The double integration method is systematic, Macaulay's method is efficient for beams with multiple loads, and the moment-area method provides quick approximations.

Fill in the blank: The method that uses step functions to handle discontinuous loading conditions is called ________ method.

Box 3.2
Methods of Calculating Beam Deflection

1. Double Integration Method

- Derives directly from the Euler-Bernouli bending equation: $\frac{EI^2}{dx^2} = M$
- Requires two integrations to find deflection $y(x)$.
- Boundary conditions are solve and crontegratious constants.
- Best for simditions are use solve for integration moment functions.
- Simplifies integration ate avoing multiple regions.

2. Maculay's Method

- Use sindicatiry of functions (Macuulay fMataullay brackets) to handle discondious loads.
- Allows a single expression for $M(x)$ across a the a entire beam multiple regions.

3. Moment-Area Method

- Geometric method based the area the area of M/EI diagram.
- Two theormes relate tangents, slope, and deflection at specific points.
- Useful Doest require integration to find the deflection function $y(x)$

3.2.3

Practical applications in beam design

Calculating slopes and deflections is essential in beam design because excessive deflection can affect the usability and safety of structures. For example, a floor beam that deflects too much may cause discomfort to occupants or damage finishes. Engineers therefore check both strength and serviceability criteria when designing beams.

Fill in the blank: Excessive deflection in a beam can affect both safety and ________ of a structure.

Plate 3.1 Beam deflection in a building floor

Pilot questions 3.2

1. State the mathematical relationship between bending moment and deflection.
2. Which method of deflection calculation uses step functions?
3. Name one graphical method used to calculate beam deflection.
4. Why is it important to check deflection in beam design?
5. Give one example of a practical consequence of excessive beam deflection.

3.3 Analysis Of Portal Frames

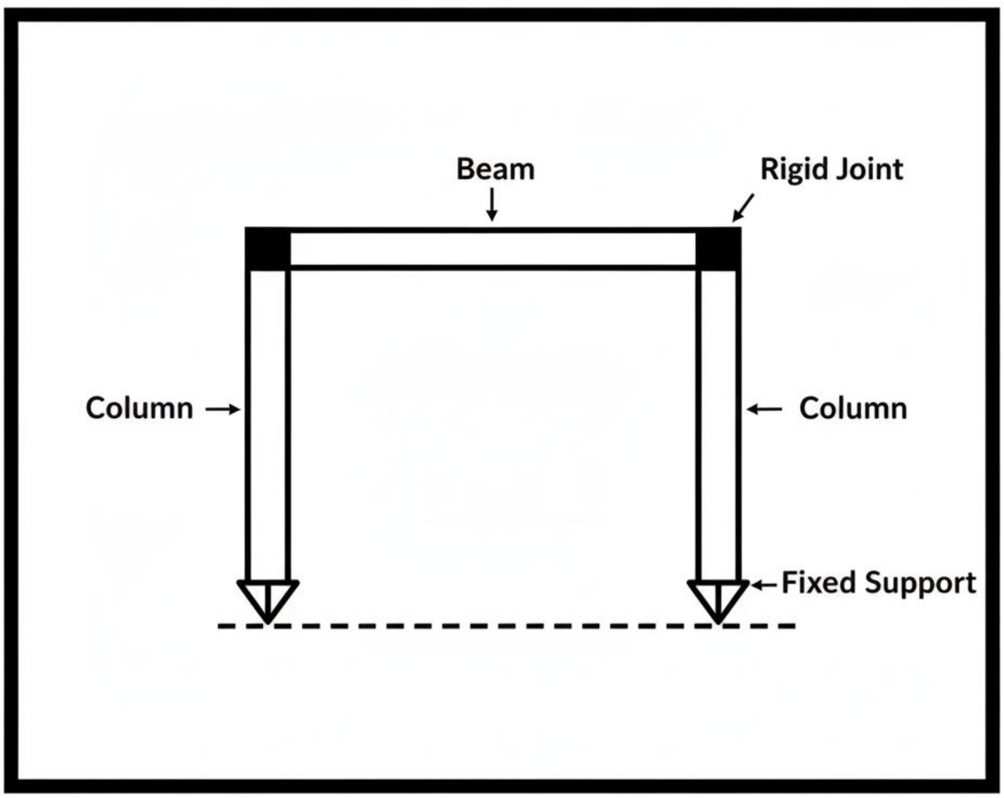
3.3.1 Structural behaviour of portal frames

A **portal frame** is a rigid structural system composed of vertical columns and horizontal beams connected by rigid joints. Unlike simple frames, portal frames resist both vertical and lateral loads due to their rigidity. They are widely used in warehouses, industrial buildings, and large-span structures because they provide stability without the need for extensive bracing.

The behaviour of portal frames under load is characterised by bending in the beams and columns, shear forces at the joints, and deflections that occur due to the applied loads. Engineers must carefully analyse these behaviours to ensure the frame can safely resist applied forces.

Fill in the blank: A portal frame is a rigid structural system composed of vertical columns and ________.

Figure 3.4 Basic portal frame structure



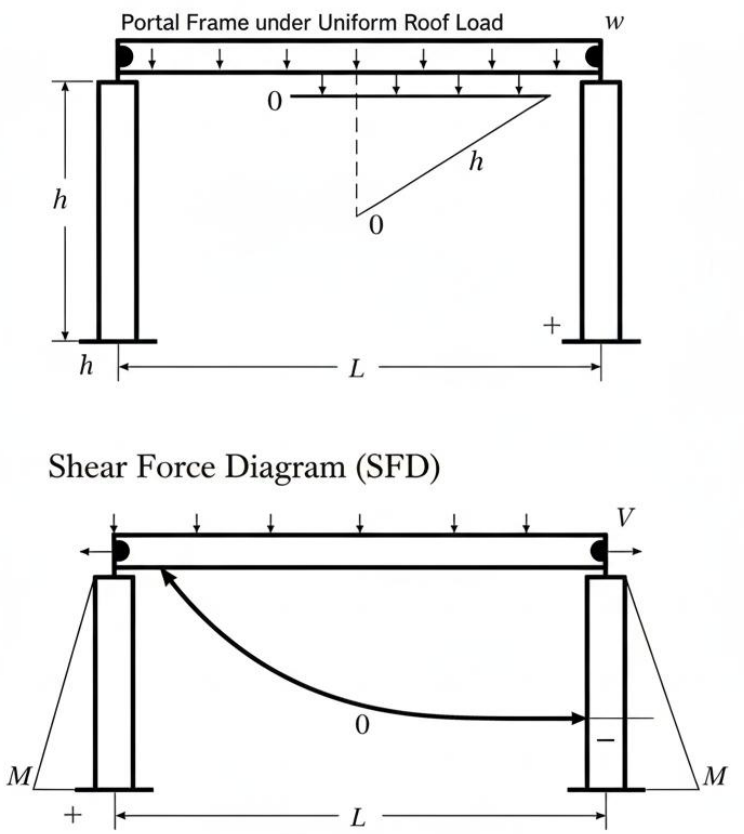
3.3.2 Shear forces and bending moments in portal frames

Portal frames experience both **shear forces** and **bending moments** when subjected to loads. The distribution of these forces depends on the type of loading and the rigidity of the joints. For example, under uniform roof loads, the beams experience bending while the columns resist both axial and bending forces.

Shear force diagrams and bending moment diagrams for portal frames are more complex than those for simple beams, but they follow the same principles of equilibrium. Engineers use these diagrams to identify critical sections where reinforcement may be required.

Fill in the blank: In portal frames, columns resist both axial forces and ________ forces.

Figure 3.5 Shear force and mear bending moment diagrams for a a portal frame



3.3.3 Deflection analysis of portal frames

Deflection analysis is critical in portal frames because excessive deflection can compromise the stability of the structure. Deflections occur due to bending in the beams and columns, and they are influenced by the stiffness of the joints and the geometry of the frame.

Methods such as the moment distribution method or computer-based finite element analysis are often used to calculate deflections in portal frames. Engineers must ensure that deflections remain within acceptable limits to maintain both safety and serviceability.

Fill in the blank: Excessive deflection in portal frames can compromise both stability and ________ of the structure.

Box 3.3 Methods of Deffection Analysis in Portal Frames

1. Moment Distribution Method

- Iterative technique for indertmeate structures.
- Distribute fixed-end moments to achieve equilblium.

2. Finite Element Analysis (FEA)

- Numerical method suitable complex geomerties.
- Divides the structure into small elements (mesh) and solves equations for each node.

Pilot questions 3.3

1. Define a portal frame and state one of its common applications.
2. What types of forces do columns in portal frames resist?
3. Why are shear force and bending moment diagrams important in portal frame analysis?
4. Name one method used to calculate deflections in portal frames.
5. What structural issue can arise if portal frame deflections are excessive?

Series Summary

This Series has focused on the analysis of beams and portal frames, emphasising how shear forces, bending moments, slopes, and deflections influence structural behaviour. The purpose has been to equip you with the analytical tools needed to evaluate these responses so that beams and frames can be designed safely and effectively.

We began by defining shear force and bending moment, then explored how to construct diagrams under different loading conditions. Following that, we examined the relationship between bending moment and deflection, applying methods such as double integration and Macaulay's approach to calculate slopes and deflections. Finally, we analysed portal frames, considering their structural behaviour, internal forces, and deflections. By completing this Series, you should now be able to define and construct shear force and bending moment diagrams, apply methods to calculate beam deflections, and analyse the behaviour of portal frames in line with the Series Learning Outcomes (SLOs 3.1 to 3.6).

Glossary of Terms

- **Axial force**: A force that acts along the length of a structural member, either stretching it in tension or compressing it.
- **Bending moment**: The internal turning effect within a beam or frame caused by external loads, which tends to bend the member.
- **Beam**: A long, straight structural member designed to resist loads applied perpendicular to its length.
- **Column**: A vertical structural member that primarily resists compression and supports beams or frames.
- **Deflection**: The displacement or bending of a structural member under load, measured from its original position.
- **Joint**: The point where two or more structural members are connected, often assumed to be pinned in truss analysis.
- **Moment of inertia (I)**: A property of a beam's cross-section that indicates its resistance to bending.
- **Modulus of elasticity (E)**: A material property that measures stiffness, showing how much it resists deformation under stress.
- **Portal frame**: A rigid structural system made of beams and columns connected by rigid joints, commonly used in large-span buildings.
- **Shear force**: The internal force in a beam or frame that acts parallel to its cross-section, tending to slide one part against another.
- **Shear force diagram (SFD)**: A graphical representation showing how shear force varies along the length of a beam or frame.
- **Slope**: The angle of deflection or rotation of a beam at a particular point under load.
- **Statical determinacy**: A condition where all internal forces and reactions in a structure can be calculated using only equilibrium equations.
- **Truss**: A structural framework made of straight members connected at joints, usually arranged in triangular units to provide stability.

- **Uniformly distributed load (UDL)**: A load that is spread evenly along the length of a beam, producing a linear variation in shear force.

Concept Check Questions 3

Objective questions

1. Define shear force in a beam. (Linked to SLO 3.1)
2. Which diagram shows how bending moment varies along the span of a beam? (Linked to SLO 3.2)
3. Identify the method that uses step functions to handle discontinuous loading conditions. (Linked to SLO 3.4)
4. A portal frame is composed of vertical columns and ________. (Linked to SLO 3.5)
5. Which type of load produces a linear variation in shear force? (Linked to SLO 3.2)

Short-answer questions

6. Explain why shear force and bending moment diagrams are important in beam analysis. (Linked to SLO 3.2)
7. State one practical consequence of excessive beam deflection. (Linked to SLO 3.4)
8. Describe the structural behaviour of portal frames under uniform roof loads. (Linked to SLO 3.5)
9. Name one graphical method used to calculate beam deflection. (Linked to SLO 3.4)
10. What structural issue can arise if portal frame deflections are excessive? (Linked to SLO 3.6)

Long-answer questions

11. Analyse the relationship between bending moment and deflection in beams, explaining how material and geometry influence this relationship. (Linked to SLO 3.3)
12. Evaluate the advantages and limitations of using portal frames in large-span structures, with reference to their behaviour under load. (Linked to SLO 3.5, SLO 3.6)

Answers to Concept Check Questions 3

Objective questions

1. Shear force is the internal force in a beam that acts parallel to its cross-section.
2. Bending moment diagram (BMD).
3. Macaulay's method.
4. Beams.
5. Uniformly distributed load (UDL).

Short-answer questions

6. They show how shear force and bending moment vary along the span, helping identify critical sections for design.
7. Excessive deflection can cause discomfort to occupants or damage finishes.
8. Under uniform roof loads, beams bend while columns resist both axial and bending forces.
9. Moment-area method.
10. Excessive deflection can compromise both stability and serviceability.

Long-answer questions

11.

- **Basic response**: The bending moment is directly related to deflection through the equation d2ydx2=MEI\frac{d^2y}{dx^2} = \frac{M}{EI}. This shows that deflection depends on the bending moment, the material's modulus of elasticity, and the cross-sectional moment of inertia.
- **Value-added response**: Beyond the basics, learners may note that materials with higher stiffness (greater EE) and sections with larger II reduce deflection. They may also discuss how engineers use this relationship to design beams that balance strength and serviceability, ensuring both safety and comfort in structures.

12.

- **Basic response**: Portal frames provide stability and resist both vertical and lateral loads, making them suitable for large-span structures. However, their analysis is more complex, and excessive deflection can be a limitation.
- **Value-added response**: Outstanding learners may add that portal frames reduce the need for internal supports, allowing open spaces in warehouses and industrial buildings. They may also discuss how modern finite element methods enhance portal frame analysis, enabling engineers to optimise designs for both economy and performance.

Answers to Pilot Questions 3

Part 3.1

1. Shear force is the internal force in a beam that acts parallel to its cross-section.
2. A bending moment diagram is used to show how bending moment varies along the span of a beam.
3. A uniformly distributed load produces a linear variation in shear force.
4. A point load causes a sudden jump in the shear force diagram.
5. Point loads act at a single point, while uniformly distributed loads are spread evenly along the beam.

Part 3.2

1. The relationship is given by d2ydx2=MEI\frac{d^2y}{dx^2} = \frac{M}{EI}, showing how bending moment relates to deflection.
2. Macaulay's method uses step functions to handle discontinuous loading conditions.
3. The moment-area method is a graphical technique used to calculate beam deflection.
4. Checking deflection ensures beams meet serviceability requirements and remain safe for use.
5. Excessive deflection can cause discomfort to occupants or damage finishes in a structure.

Part 3.3

1. A portal frame is a rigid structural system of beams and columns, commonly used in warehouses and industrial buildings.
2. Columns in portal frames resist both axial forces and bending forces.
3. Shear force and bending moment diagrams identify critical sections and guide reinforcement design.
4. The moment distribution method is one technique used to calculate deflections in portal frames.
5. Excessive deflection can compromise both the stability and serviceability of the structure.

References and Attributions 3

Textual references

- Hibbeler, R. C. (2017). *Structural Analysis* (9th ed.). Pearson Education.
- Ghali, A., Neville, A. M., & Brown, T. G. (2014). *Structural Analysis: A Unified Classical and Matrix Approach* (6th ed.). CRC Press.
- Open Educational Resources (OER Commons). (n.d.). Structural mechanics resources. Retrieved from https://www.oercommons.org
- MIT OpenCourseWare. (n.d.). *Structural Engineering and Mechanics*. Retrieved from https://ocw.mit.edu

Illustrations

- *Figure 3.1 Shear force and bending moment in a simply supported beam*: AI-generated using Copilot image generation. Prompt: "Generate a diagram of a simply supported beam with a point load, showing shear force and bending moment arrows."
- *Figure 3.2 Shear force and bending moment diagrams for a uniformly loaded beam*: AI-generated using Copilot image generation. Prompt: "Generate shear force and bending moment diagrams for a simply supported beam with a uniformly distributed load."
- *Box 3.1 Common loading cases for beams*: Adapted from OER Commons structural mechanics resources.
- *Figure 3.3 Relationship between bending moment and deflection in a beam*: AI-generated using Copilot image generation. Prompt: "Generate a diagram of a simply supported beam under load showing bending moment and deflection curve."

- *Box 3.2 Methods of calculating beam deflection*: Adapted from MIT OpenCourseWare structural mechanics notes.
- *Plate 3.1 Beam deflection in a building floor*: Sourced from Wikimedia Commons (Creative Commons licence). Search string: "OER photo beam deflection building floor."
- *Figure 3.4 Basic portal frame structure*: AI-generated using Copilot image generation. Prompt: "Generate a diagram of a portal frame with labelled beams, columns, and rigid joints."
- *Figure 3.5 Shear force and bending moment diagrams for a portal frame*: AI-generated using Copilot image generation. Prompt: "Generate shear force and bending moment diagrams for a portal frame under uniform roof load."
- *Box 3.3 Methods of deflection analysis in portal frames*: Adapted from OER Commons and MIT OpenCourseWare resources.

Attribution notes

- All AI-generated figures were created using Copilot's image generation tool based on prompts provided in the instructional content.
- External images and diagrams sourced from Wikimedia Commons and OER Commons are used under Creative Commons licences, ensuring open educational use.
- Textual references are provided in APA style for clarity and consistency.

Series 4: Influence Lines and Applications to Moving Loads

Proposed Outline for Series 4

- **Part 4.1 Fundamentals of Influence Lines**
    - 4.1.1 Definition of influence lines
    - 4.1.2 Construction of influence lines for reactions
    - 4.1.3 Construction of influence lines for shear force and bending moment
- **Part 4.2 Applications of Influence Lines to Moving Loads**
    - 4.2.1 Determining maximum shear and bending moment under moving loads
    - 4.2.2 Influence lines for trusses and frames
    - 4.2.3 Practical examples in bridges and roadways
- **Part 4.3 Engineering Significance of Influence Lines**
    - 4.3.1 Importance in structural design and safety
    - 4.3.2 Limitations of influence line analysis
    - 4.3.3 Transition to advanced methods for indeterminate structures

Introduction

When vehicles move across bridges or trains pass over railway tracks, the loads they carry do not remain fixed at one point. Instead, these loads travel along the structure, creating varying effects at different positions. Engineers must be able to predict how forces such as shear and bending moments change as loads move, and this is where **influence lines** become essential. Influence lines provide a powerful tool for analysing the behaviour of structures under moving loads, making them highly relevant to the study of beams, trusses, and frames in structural mechanics.

The aim of this Series is to introduce you to the principles of influence lines and to equip you with the skills needed to construct and apply them in the analysis of moving loads on structures.

We shall commence this Series by examining the fundamentals of influence lines, including their definition and construction for reactions, shear forces, and bending moments. Next, we will move on to applications of influence lines to moving loads, where you will learn how to determine maximum effects and apply the method to trusses and frames with practical examples. Finally, we will wrap up by considering the engineering significance of influence lines, discussing their importance in design, their limitations, and how they lead into advanced methods for indeterminate structures.

Series Learning Outcomes

Upon completing this Series, you should be able to:

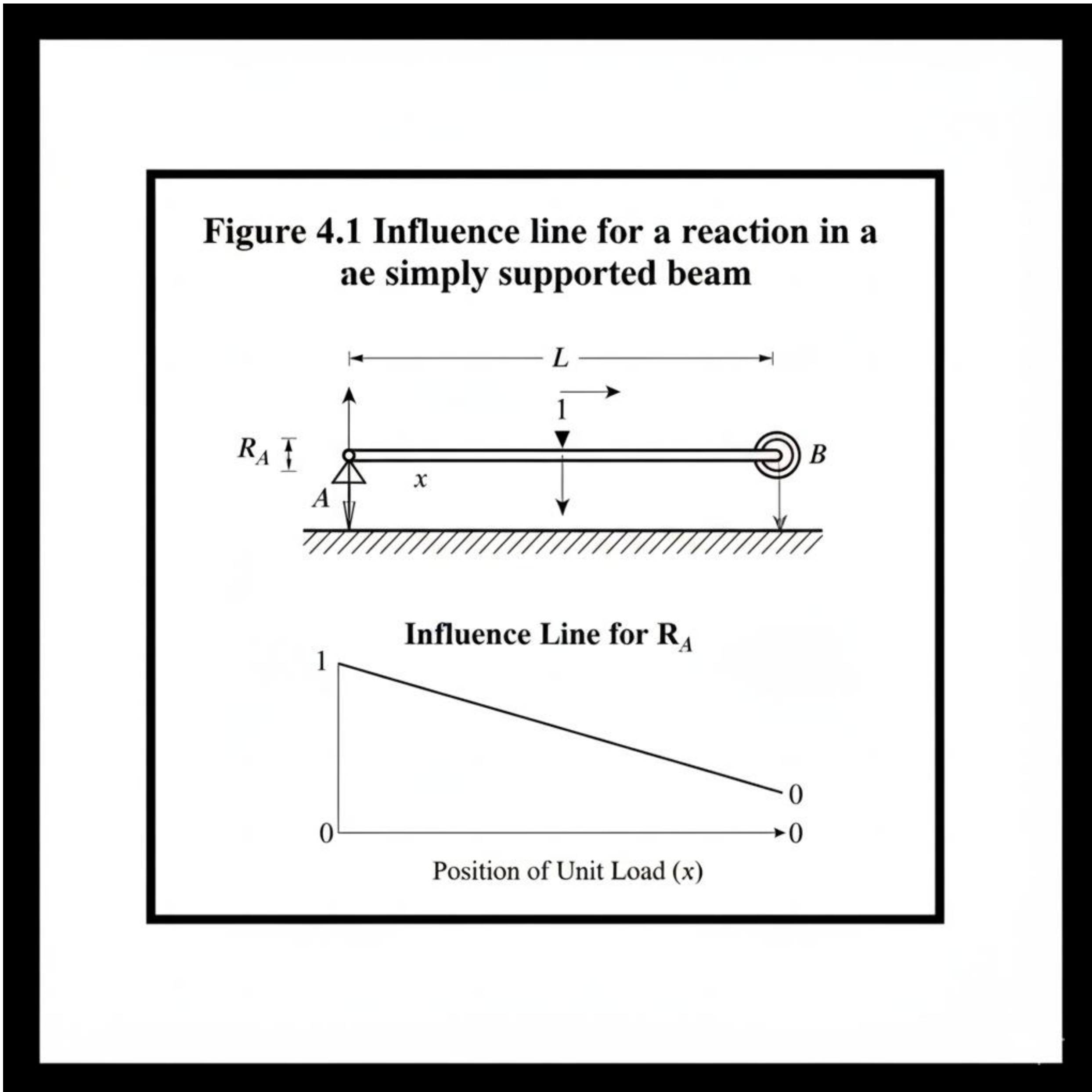
- **SLO 4.1** define the concept of **influence lines** and explain how they are constructed for reactions, shear forces, and bending moments,
- **SLO 4.2** demonstrate the application of influence lines to determine maximum shear and bending moments under moving loads,
- **SLO 4.3** apply influence line methods to trusses and frames, showing their relevance in practical structural analysis, and
- **SLO 4.4** evaluate the engineering significance of influence lines, including their importance in design, limitations, and connection to advanced methods for indeterminate structures.

4.1 Fundamentals Of Influence Lines

4.1.1 Definition of influence lines

An **influence line** is a graphical representation that shows how a particular response in a structure, such as a reaction, shear force, or bending moment, varies as a single unit load moves across the structure. Influence lines are essential in structural mechanics because they allow engineers to determine the most critical positions of moving loads, such as vehicles on bridges or trains on tracks.

Fill in the blank: An influence line shows how a structural response varies as a ________ load moves across the structure.

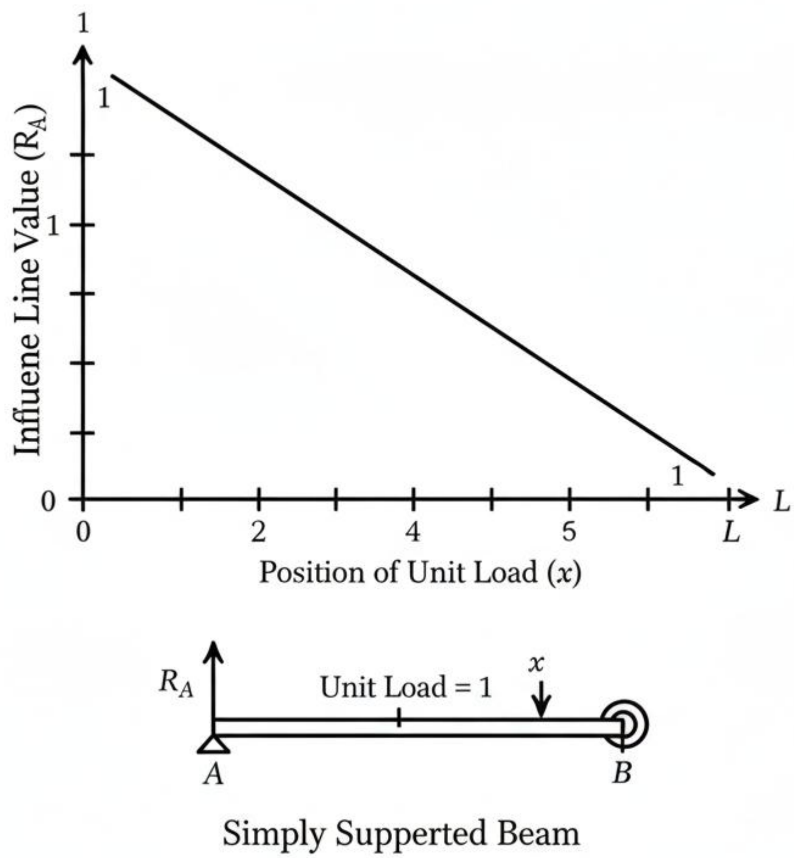


4.1.2 Construction of influence lines for reactions

To construct an influence line for a reaction, a unit load is placed at different points along the span of the structure, and the corresponding reaction is calculated using equilibrium equations. Plotting these values against the position of the load produces the influence line. For a simply supported beam, the influence line for a reaction is a straight line that decreases linearly from one support to the other.

Fill in the blank: The influence line for a reaction in a simply supported beam is a ________ line.

Figure 4.2 Linear influence line for a support reaction

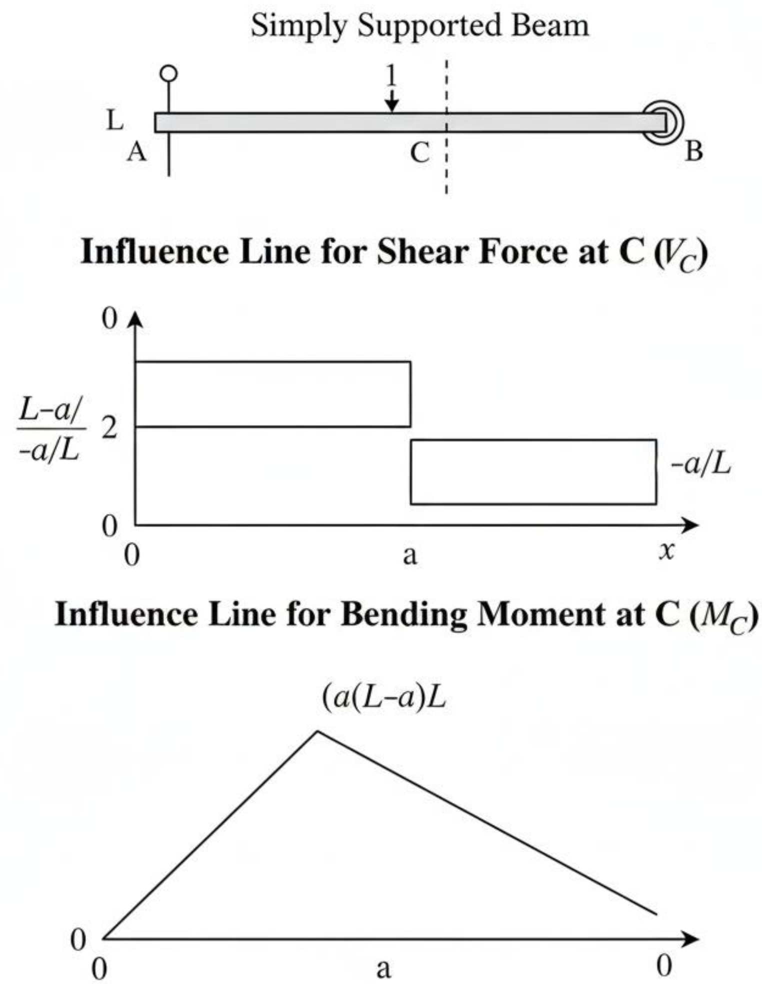


4.1.3 Construction of influence lines for shear force and bending moment

Influence lines can also be constructed for **shear force** and **bending moment** at specific points in a structure. For shear force, the influence line shows how the shear at a chosen section changes as the unit load moves across the span. For bending moment, the influence line shows how the moment at a chosen section varies with the position of the load. These diagrams are particularly useful in identifying the position of moving loads that produce maximum shear or bending moment.

Fill in the blank: Influence lines for bending moment show how the moment at a chosen section varies with the ________ of the load.

Figure 4.3 Influence lines for shear force and bending moment at a section



Pilot questions 4.1

1. Define an influence line.
2. What does an influence line for a reaction in a simply supported beam look like?
3. How is an influence line for a reaction constructed?
4. What structural responses can influence lines represent apart from reactions?
5. Why are influence lines useful in analysing moving loads?

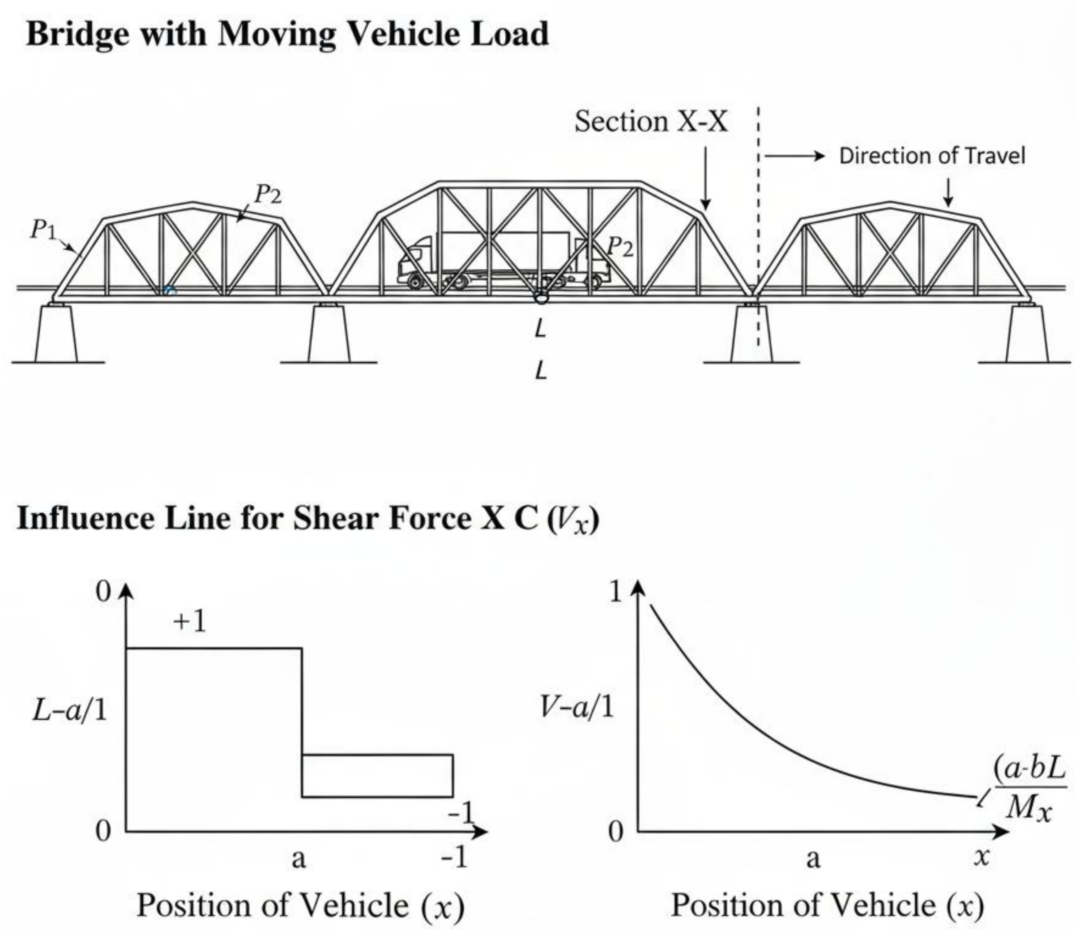
4.2 Applications Of Influence Lines To Moving Loads

4.2.1 Determining maximum shear and bending moment under moving loads

Influence lines are particularly useful when analysing structures subjected to **moving loads**, such as vehicles on bridges. By plotting the influence line for shear force or bending moment at a specific section, engineers can determine the position of the moving load that produces the maximum effect. This allows for efficient design, ensuring that the structure can safely carry the most critical loading scenario.

Fill in the blank: Influence lines help identify the position of a moving ________ that produces maximum shear or bending moment.

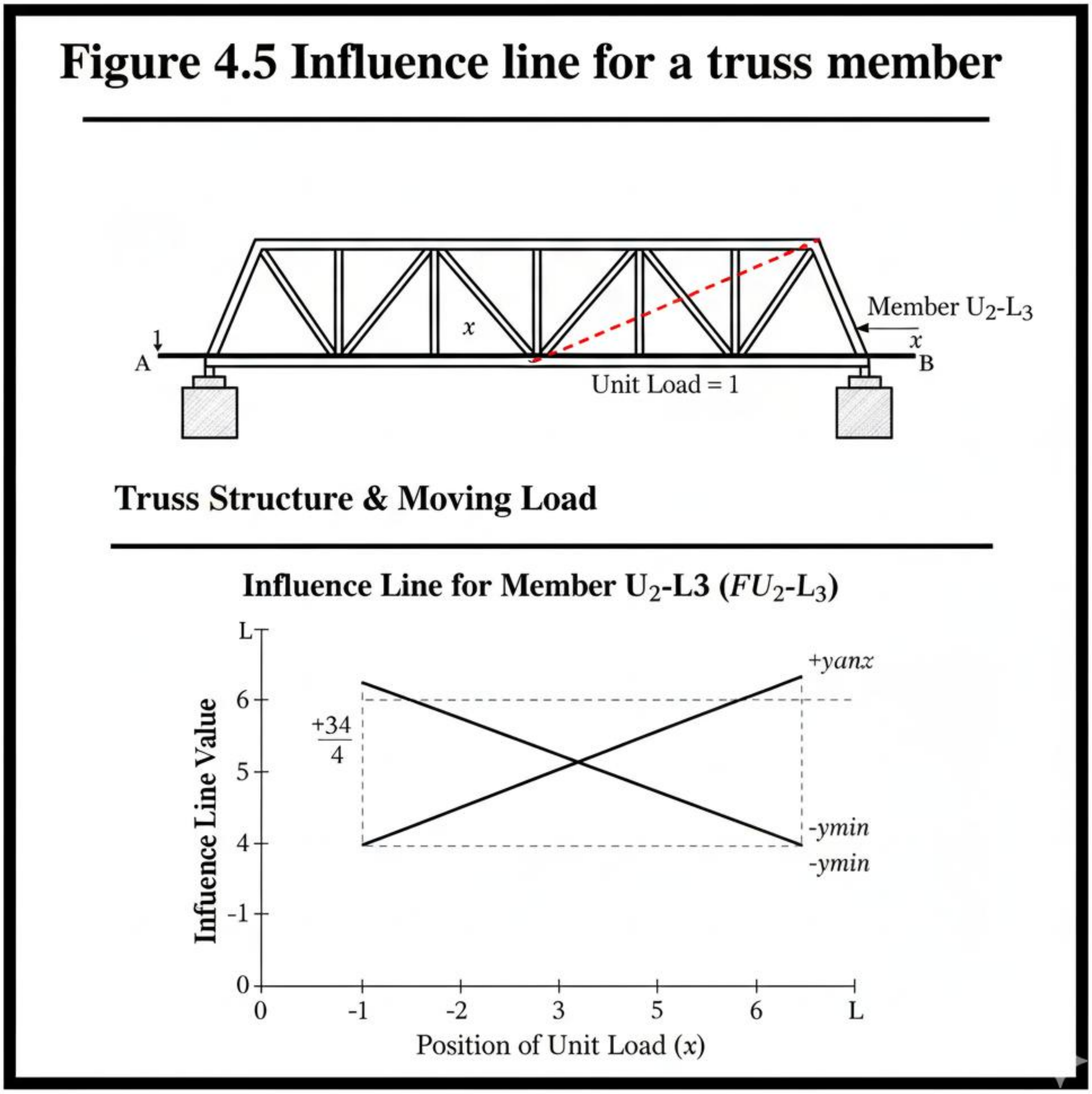
Figure 4.4 Influence lines applied to moving loads on a bridge



4.2.2 Influence lines for trusses and frames

Influence lines can also be applied to **trusses** and **frames**, which are common in bridges and buildings. For trusses, influence lines show how the force in a particular member changes as a unit load moves across the structure. For frames, they illustrate how reactions, shear forces, and bending moments vary with load position. These applications are vital in identifying critical members and sections that must be designed to withstand maximum forces.

Fill in the blank: Influence lines for trusses show how the force in a particular ________ changes as a load moves.



4.2.3 Practical examples in bridges and roadways

In practice, influence lines are widely used in the design of **bridges** and **roadways**. Engineers use them to determine the maximum effects of moving vehicles, ensuring that the structure is both safe and economical. Influence lines help identify the worst-case scenarios, such as the position of heavy trucks on a bridge span, and guide the placement of reinforcement or structural supports.

Fill in the blank: Engineers use influence lines in bridge design to determine the worst-case scenario caused by heavy ________.

Plate 4.1 Practical application of influence lines in bridge design

Pilot questions 4.2

1. How do influence lines help in determining maximum shear and bending moment under moving loads?
2. What structural responses can influence lines represent in trusses?
3. Why are influence lines important in the analysis of frames?
4. Give one practical example of how influence lines are applied in bridge design.
5. What type of scenario do influence lines help engineers identify in roadway structures?

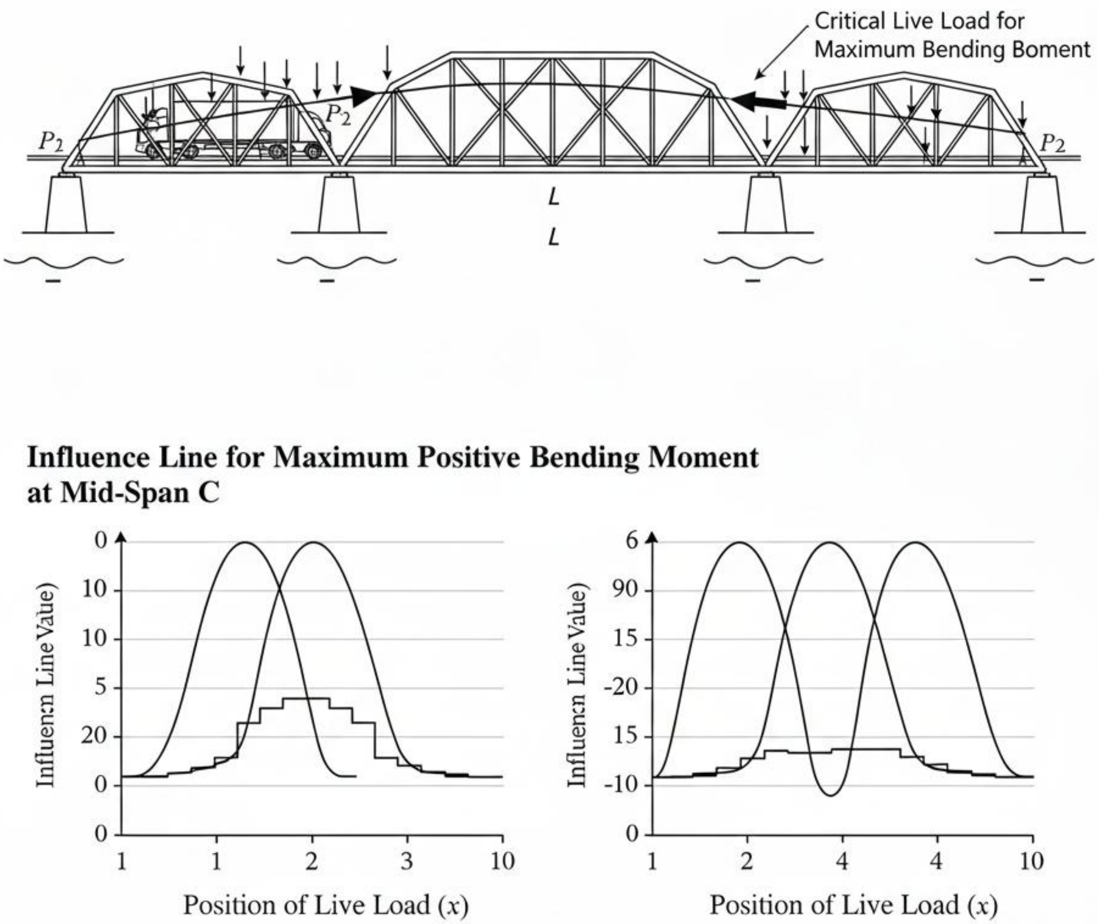
4.3 Engineering Significance Of Influence Lines

4.3.1 Importance in structural design and safety

Influence lines are vital in **structural design** because they allow engineers to identify the most critical positions of moving loads. This ensures that structures such as bridges, trusses, and frames are designed to withstand maximum effects safely. By using influence lines, engineers can optimise material usage, reduce costs, and enhance safety margins. They are particularly important in serviceability checks, ensuring that deflections and stresses remain within acceptable limits.

Fill in the blank: Influence lines help engineers identify the most ________ positions of moving loads in a structure.

Figure 4.6 Importance of influence lines in bridge design



4.3.2 Limitations of influence line analysis

Although influence lines are powerful tools, they have limitations. They are most effective for statically determinate structures and become less straightforward when applied to indeterminate structures. Additionally, influence lines are based on linear elastic behaviour, which may not fully capture real-world complexities such as material nonlinearity or dynamic effects. Engineers must therefore use them alongside other methods to achieve accurate results.

Fill in the blank: Influence lines are most effective for ________ determinate structures.

Box 4.1 Limitations of influence line analysis

Limitations

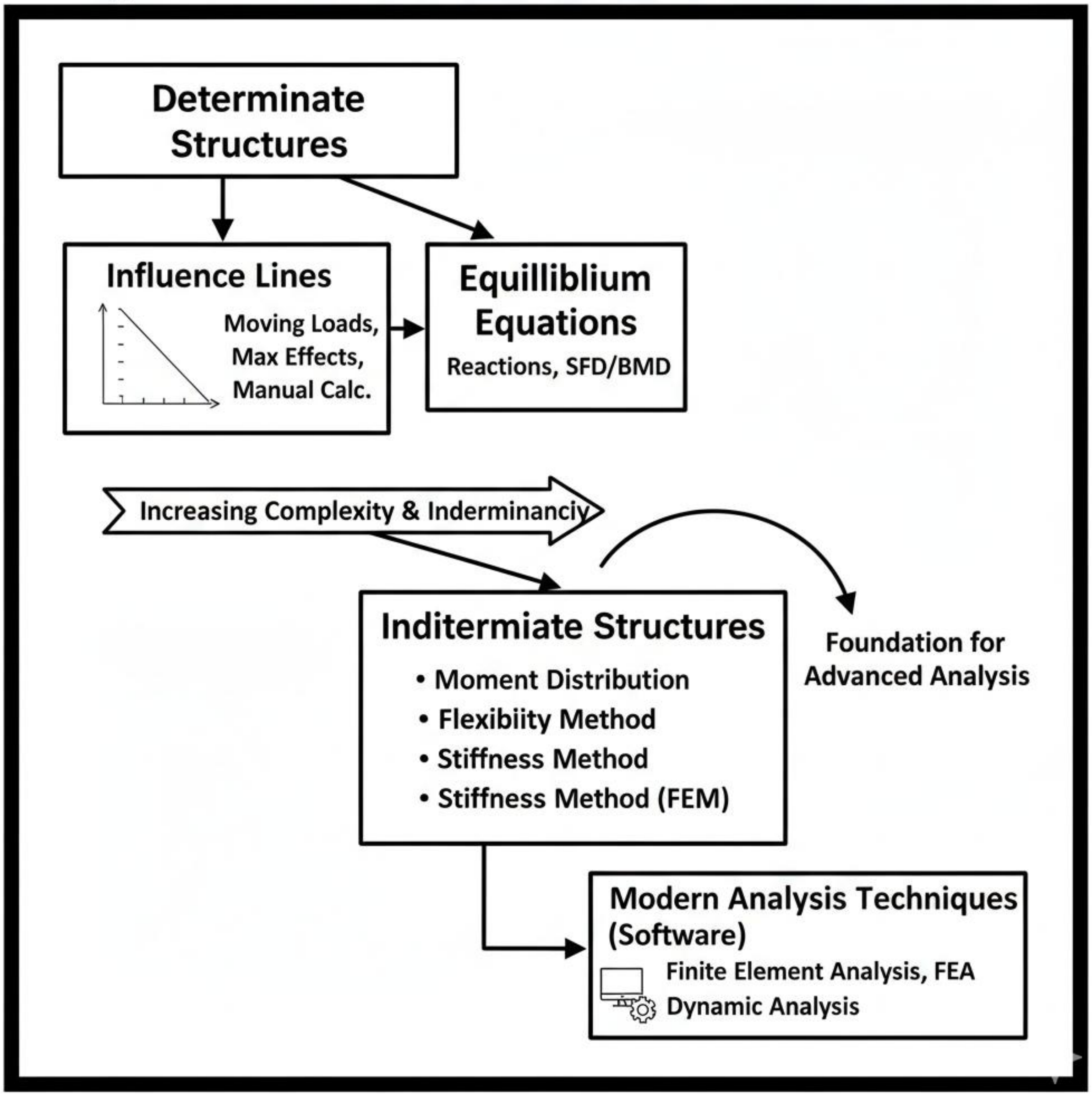
1. Primarly for statiaicly detemitate structures.
2. Can be complex for large inderdiitate structures or multiple concentrated loads.
3. Provides maximum effects for single-lane loads; combination with other lanes or wind loads requires further analysis.
4. Assumes linear-elastic material behavior.
5. Limited to influence of load position; doest directly provide deflections or stresses.

4.3.3 Transition to advanced methods for indeterminate structures

Influence lines provide a foundation for more advanced methods used in analysing **statically indeterminate structures**. While they are sufficient for determinate cases, indeterminate structures require additional techniques such as the force method, displacement method, or computer-based matrix analysis. Understanding influence lines prepares learners to transition smoothly into these advanced topics, ensuring continuity in structural mechanics education.

Fill in the blank: Indeterminate structures require additional techniques such as the ________ method or displacement method.

Figure 4.7 Transition from influence lines to advanced methods



Pilot questions 4.3

1. Why are influence lines important in structural design?
2. What is one limitation of influence line analysis?
3. For what type of structures are influence lines most effective?
4. What assumptions underlie influence line analysis?
5. Which advanced methods are required for indeterminate structures?

Series Summary

This Series has focused on the concept of influence lines and their role in analysing structures subjected to moving loads. Influence lines are essential tools in structural mechanics, enabling engineers to determine critical positions of loads and design safe, efficient structures such as bridges and frames. Their relevance lies in bridging the gap between theoretical analysis and practical applications in structural design.

We began by exploring the fundamentals of influence lines, including their definition and construction for reactions, shear forces, and bending moments. We then moved on to applications, where influence lines were used to determine maximum effects under moving loads and extended to trusses and frames with practical examples. Finally, we considered their engineering significance, highlighting their importance in design, their limitations, and how they prepare learners for advanced methods in indeterminate analysis. By achieving the Series Learning Outcomes, you should now be able to define and construct influence lines, demonstrate their application to moving loads, apply them to trusses and frames, and evaluate their significance in structural design and safety.

Glossary of Terms

bending moment The turning effect produced when a force causes a beam or structural member to bend. It is measured at a specific point along the span.

frame A structural system made up of beams and columns connected together, often used in buildings and bridges to resist loads.

influence line A graph that shows how a structural response, such as a reaction, shear force, or bending moment, changes as a unit load moves across the structure.

moving load A load that changes position along a structure, such as vehicles on a bridge or trains on a track.

reaction The force or moment provided by a support to keep a structure in equilibrium when loads are applied.

shear force The internal force in a beam or structural member that acts parallel to its cross-section, tending to slide one part of the section against another.

statically determinate structure A structure that can be fully analysed using only the equations of equilibrium, without needing additional conditions.

statically indeterminate structure A structure with more unknown forces than equilibrium equations, requiring extra conditions such as deformation compatibility for analysis.

strain energy The energy stored in a structure when it deforms under applied loads.

truss A framework made up of straight members connected at joints, commonly arranged in triangular units to provide stability and rigidity.

unit load method A deflection theorem where a fictitious unit load is applied at a point to calculate the displacement at that location.

Concept Check Questions 4

Objective Questions

1. Define an influence line. (Linked to SLO 4.1)
2. The influence line for a reaction in a simply supported beam is: a) Curved b) Linear c) Parabolic d) Constant (Linked to SLO 4.1)
3. Influence lines are most effective for which type of structures? (Linked to SLO 4.4)
4. Which method is required for analysing indeterminate structures beyond influence lines? (Linked to SLO 4.4)

Short-Answer Questions

5. Explain how influence lines help determine maximum shear force under moving loads. (Linked to SLO 4.2)
6. State one practical application of influence lines in bridge design. (Linked to SLO 4.2)
7. What limitation restricts the use of influence lines in indeterminate structures? (Linked to SLO 4.4)
8. Describe how influence lines can be applied to trusses. (Linked to SLO 4.3)

Long-Answer Questions

9. Demonstrate the construction of an influence line for bending moment at mid-span of a simply supported beam. (Linked to SLO 4.1)
   - **Basic response:** Use Series-only knowledge to describe the step-by-step process of placing a unit load, calculating bending moment values, and plotting them.
   - **Value-added response:** Extend by discussing how the influence line can be used to determine maximum bending moment under different moving load scenarios, with reference to bridge design.
10. Evaluate the engineering significance of influence lines in structural design and safety. (Linked to SLO 4.4)

- **Basic response:** Summarise their importance in identifying critical load positions, ensuring safety, and optimising material use.
- **Value-added response:** Go further by comparing influence line analysis with modern computational methods, highlighting how influence lines remain relevant as a teaching and design tool despite advances in software.

Answers to Concept Check Questions 4

Objective Questions

1. An influence line is a graph showing how a structural response varies as a unit load moves across the structure.
2. b) Linear
3. Statically determinate structures
4. Force method or displacement method

Short-Answer Questions

5. Influence lines show how shear force at a section changes as a load moves, helping identify the position of the load that produces maximum shear.
6. They are used to determine the worst-case scenario caused by heavy vehicles on a bridge span.
7. Influence lines rely on equilibrium and linear elasticity, making them less effective for indeterminate structures.
8. Influence lines for trusses show how the force in a specific member changes as a unit load moves across the structure.

Long-Answer Questions

9.

- **Basic response:** To construct an influence line for bending moment at mid-span of a simply supported beam, place a unit load at different positions along the span. Calculate the bending moment at mid-span for each position using equilibrium equations. Plot these values against the load position to obtain the influence line.
- **Value-added response:** Beyond construction, the influence line can be used to determine maximum bending moment under moving loads, such as vehicles on a bridge. Engineers use this to design reinforcement and supports, ensuring safety and efficiency.

10.

- **Basic response:** Influence lines are significant because they help identify critical load positions, ensure safety, and optimise material use in structures.
- **Value-added response:** While modern computational methods provide faster analysis, influence lines remain a valuable teaching and design tool. They offer clear visualisation of load effects and complement advanced methods, ensuring engineers understand both theory and practice.

Answers to Pilot Questions 4

Part 4.1

- An influence line is a graphical representation showing how a structural response varies as a unit load moves across the structure.
- The influence line for a reaction in a simply supported beam is a straight, linear line.
- It is constructed by placing a unit load at different positions along the span, calculating the reaction at each position, and plotting the values.
- Influence lines can represent reactions, shear forces, and bending moments.
- They are useful because they identify the positions of moving loads that produce maximum effects in structures.

Part 4.2

- Influence lines help determine maximum shear and bending moment by showing how these responses vary as a load moves, allowing engineers to identify the critical load position.
- In trusses, influence lines show how the force in a specific member changes as a unit load moves across the structure.
- In frames, they illustrate how reactions, shear forces, and bending moments vary with load position.
- A practical example is their use in bridge design to identify the worst-case scenario caused by heavy vehicles.
- They help engineers identify scenarios such as the position of heavy trucks that produce maximum structural effects.

Part 4.3

- Influence lines are important because they allow engineers to design structures safely by identifying critical load positions and optimising material use.
- One limitation is that they are mainly applicable to statically determinate structures and assume linear elastic behaviour.
- They are most effective for statically determinate structures.
- Influence line analysis assumes linear elasticity and equilibrium conditions.
- Advanced methods such as the force method and displacement method are required for indeterminate structures.

References and Attributions 4

This section compiles all references and illustration attributions used or suggested in **Series 4: Influence Lines for Moving Loads**. References are presented in APA style for clarity and consistency.

General References

- Hibbeler, R. C. (2017). *Structural Analysis* (9th ed.). Pearson.
- Kassimali, A. (2018). *Structural Analysis* (5th ed.). Cengage Learning.

- Open Educational Resources (OER) diagrams and summaries were suggested for influence lines, shear force, bending moment, trusses, frames, and bridge applications.

Illustrations

- *Figure 4.1 Influence line for a reaction in a simply supported beam* Attribution: Suggested OER search string "OER diagram influence line simply supported beam". AI-generated prompt: "Generate a diagram of a simply supported beam with a unit load moving across, showing the influence line for a reaction."
- *Figure 4.2 Linear influence line for a support reaction* Attribution: Suggested OER search string "OER influence line reaction beam graph". AI-generated prompt: "Generate a graph showing the influence line for a support reaction in a simply supported beam."
- *Figure 4.3 Influence lines for shear force and bending moment at a section* Attribution: Suggested OER search string "OER diagram influence line shear bending moment beam". AI-generated prompt: "Generate a diagram of a simply supported beam showing influence lines for shear force and bending moment at a section."
- *Figure 4.4 Influence lines applied to moving loads on a bridge* Attribution: Suggested OER search string "OER diagram influence lines moving loads bridge". AI-generated prompt: "Generate a diagram of a bridge with a moving vehicle load, showing influence lines for shear and bending moment."
- *Figure 4.5 Influence line for a truss member* Attribution: Suggested OER search string "OER diagram influence line truss member". AI-generated prompt: "Generate a diagram of a truss with a unit load moving across, showing influence line for a member."
- *Plate 4.1 Practical application of influence lines in bridge design* Attribution: Suggested OER search string "OER image bridge vehicles influence lines application". AI-generated prompt: "Generate an image of a roadway bridge with vehicles, illustrating the application of influence lines in practice."
- *Figure 4.6 Importance of influence lines in bridge design* Attribution: Suggested OER search string "OER diagram bridge influence lines critical load positions". AI-generated prompt: "Generate a diagram of a bridge showing critical load positions identified using influence lines."
- *Box 4.1 Limitations of influence line analysis* Attribution: Suggested OER search string "OER limitations of influence line analysis summary". AI-generated prompt: "Create a text box summarising the limitations of influence line analysis."
- *Figure 4.7 Transition from influence lines to advanced methods* Attribution: Suggested OER search string "OER diagram influence lines advanced methods indeterminate structures". AI-generated prompt: "Generate a diagram showing progression from influence lines to advanced methods for indeterminate structures."

Series 5: Deflection Theorems and Introduction to Statically Indeterminate Structures

Proposed Outline for Series 5

- **Part 5.1 Deflection Theorems**
    - 5.1.1 Statement of unit load method
    - 5.1.2 Statement of Castigliano's theorem
    - 5.1.3 Applications of deflection theorems in beams and trusses
- **Part 5.2 Introduction to Statically Indeterminate Structures**
    - 5.2.1 Definition and characteristics of indeterminate structures
    - 5.2.2 Degrees of indeterminacy
    - 5.2.3 Examples of indeterminate beams and frames
- **Part 5.3 Basic Approaches to Indeterminate Analysis**
    - 5.3.1 Force method (flexibility method)
    - 5.3.2 Displacement method (stiffness method)
    - 5.3.3 Practical relevance and limitations

Introduction

In the last Series, we examined influence lines and their applications to moving loads, which allowed us to predict how structures respond under varying conditions. Building on that knowledge, we now turn to deflection theorems and the introduction of statically indeterminate structures. These topics are crucial because they extend the scope of structural analysis beyond equilibrium alone, enabling engineers to evaluate displacements and tackle structures that cannot be solved using basic equilibrium equations.

The aim of this Series is to introduce you to the principles of deflection theorems and to provide a foundation for understanding statically indeterminate structures. By the end of this Series, you will be equipped with the skills to apply deflection methods, recognise indeterminacy, and appreciate the basic approaches used in indeterminate analysis.

We shall commence this Series by exploring deflection theorems, including the unit load method and Castigliano's theorem, and their applications in beams and trusses. Next, we will move on to the introduction of statically indeterminate structures, where we will define their characteristics, examine degrees of indeterminacy, and consider examples of indeterminate beams and frames. Finally, we will wrap up by discussing basic approaches to indeterminate analysis, focusing on the force method and displacement method, and evaluating their practical relevance and limitations. This progression will prepare you for more advanced studies in structural mechanics and design.

Series Learning Outcomes

Upon completing this Series, you should be able to:

- **SLO 5.1** state the unit load method as a deflection theorem,
- **SLO 5.2** explain Castigliano's theorem and its application to structural analysis,

- **SLO 5.3** apply deflection theorems to calculate displacements in beams and trusses,
- **SLO 5.4** define statically indeterminate structures and describe their main characteristics,
- **SLO 5.5** determine the degree of indeterminacy for given beams and frames,
- **SLO 5.6** illustrate examples of indeterminate structures in engineering practice,
- **SLO 5.7** analyse the basic principles of the force method (flexibility method),
- **SLO 5.8** demonstrate the displacement method (stiffness method) in simple cases, and
- **SLO 5.9** evaluate the practical relevance and limitations of indeterminate analysis methods.

5.1 Deflection Theorems

5.1.1 Statement of unit load method

The **unit load method** is a deflection theorem used to calculate displacements in structures. It involves applying a fictitious unit load at the point and in the direction where the displacement is required. By evaluating the work done by internal forces under this unit load, the actual displacement can be determined. This method is particularly useful for calculating deflections in beams and trusses without solving complex differential equations.

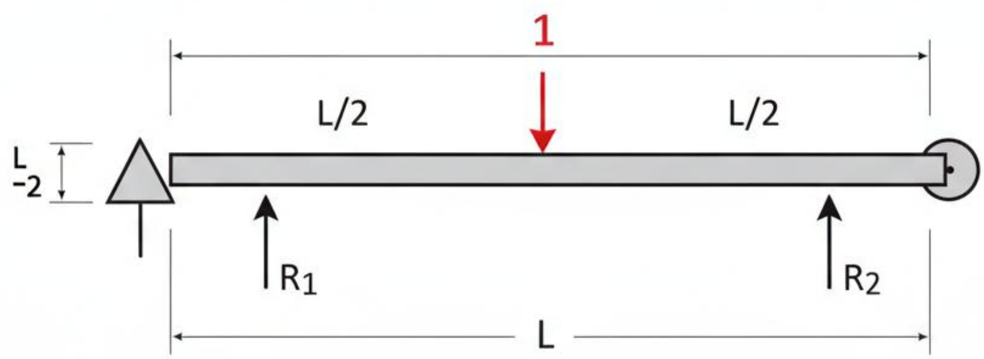


Figure 5.1 Application of unit load method to a beam

Fill in the blank: In the unit load method, a fictitious ________ load is applied at the point of interest.

5.1.2 Statement of Castigliano's theorem

Castigliano's theorem states that the partial derivative of the strain energy of a structure with respect to a load gives the displacement in the direction of that load. Strain energy is the energy stored in a structure due to deformation under applied loads. This theorem is powerful because it allows engineers to calculate displacements by considering energy principles rather than solving equilibrium equations alone.

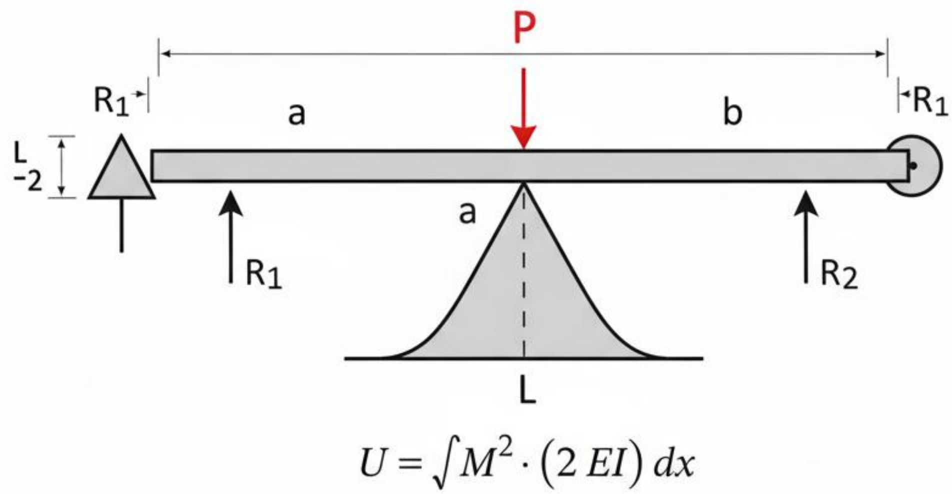


Figure 5.2 Castigllano's theorem applied to a loaded beam

Fill in the blank: Castigliano's theorem states that displacement is obtained by differentiating ________ energy with respect to the load.

5.1.3 Applications of deflection theorems in beams and trusses

Deflection theorems are widely applied in structural analysis. In beams, they help determine vertical deflections at specific points, which is critical for serviceability and safety. In trusses, they are used to calculate joint displacements, ensuring that the structure remains stable under load. These methods are especially valuable when dealing with complex structures where direct integration methods would be cumbersome.

Box 5.1 Applications of Deflection Theorems

Beams

- Finding displacement/slope at any point.
- Determing reactions in indemimate beams
- Analyzing curved beams
- Analyzing curved beams
- Effect of shear deromations
- Impact of temperature changes

Trusses

- Joint displacements
- Support settlements
- Member length changes (fabrication erros/temperature)
- Influence line for reactions/menber forces
- Determining stability of structures

Fill in the blank: Deflection theorems are especially valuable when analysing ________ structures where direct integration is difficult.

Pilot questions 5.1

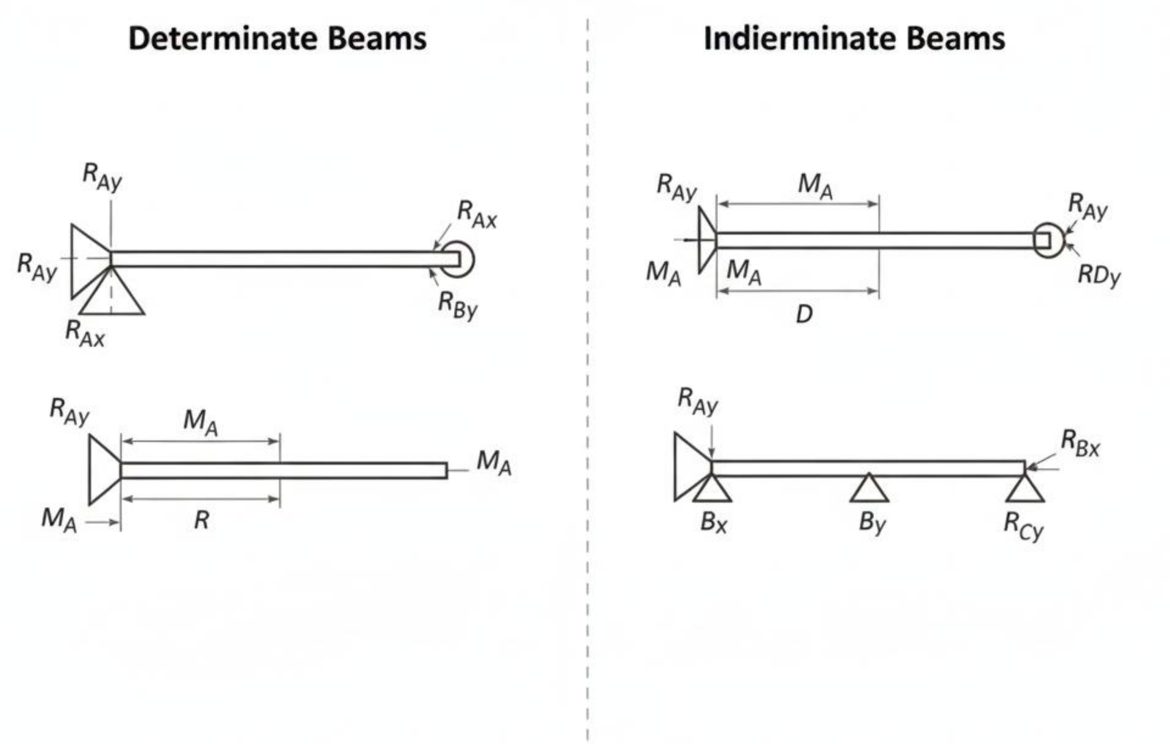
1. What is the main principle of the unit load method?
2. According to Castigliano’s theorem, how is displacement calculated?
3. Why are deflection theorems useful in truss analysis?
4. What type of energy is considered in Castigliano’s theorem?
5. Give one practical application of the unit load method in beams.

5.2 Introduction to Statically Indeterminate Structures

5.2.1 Definition and characteristics of indeterminate structures

A **statically indeterminate structure** is one in which the number of unknown reactions or internal forces exceeds the number of available equilibrium equations. Unlike determinate structures, these cannot be solved using equilibrium alone. They require additional conditions, such as compatibility of deformations, to be analysed. Indeterminate structures are often more stable and resilient because they possess redundancy, meaning they can redistribute loads if one member fails.

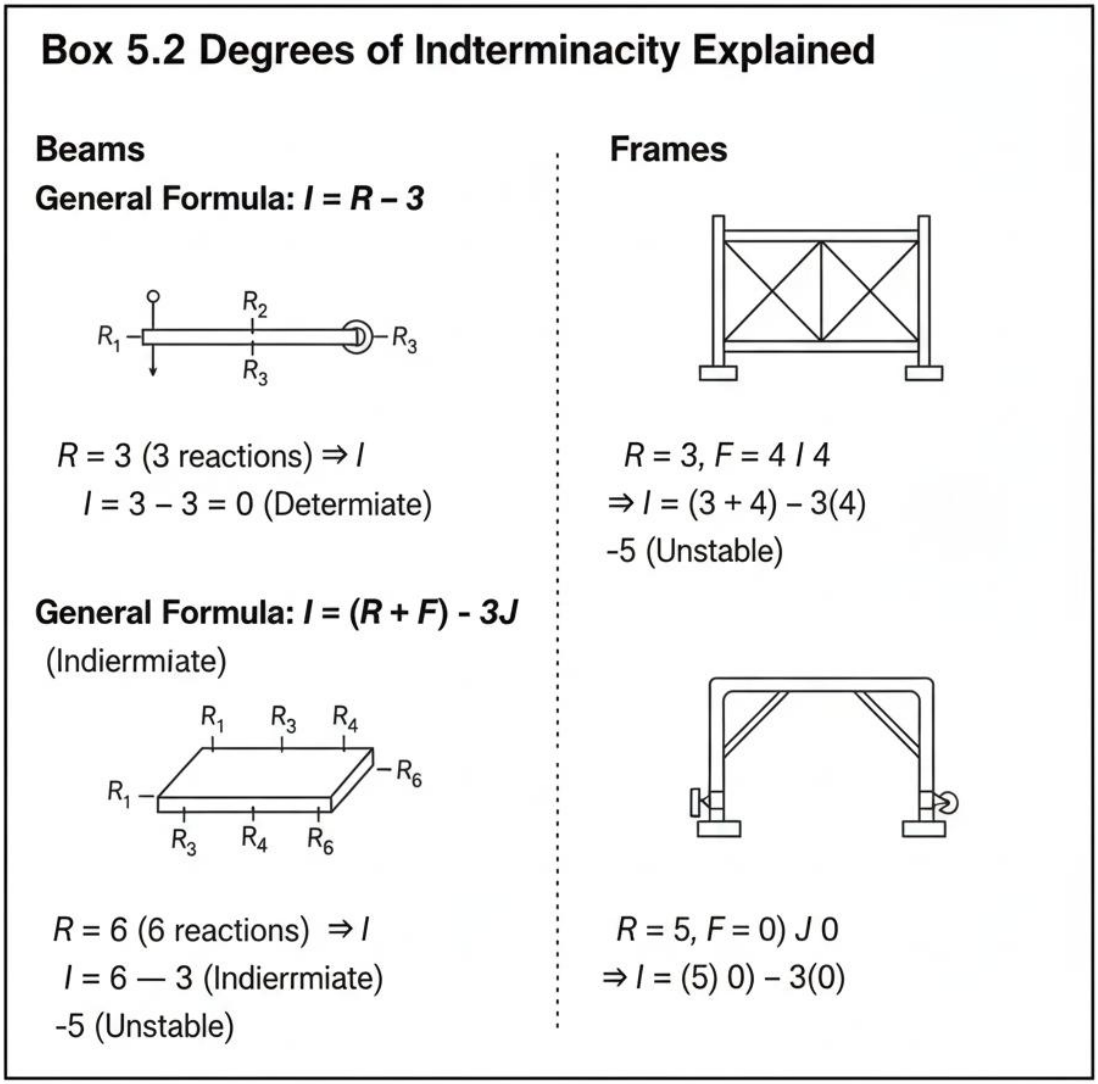
Figure 5.3 Comparison of deterinate and inderminate beams



Fill in the blank: A statically indeterminate structure has more unknown forces than available ________ equations.

5.2.2 Degrees of indeterminacy

The **degree of indeterminacy** refers to the number of extra unknowns beyond the equilibrium equations. For example, in a beam with three supports, there may be four unknown reactions but only three equilibrium equations, giving a degree of indeterminacy of one. This measure helps engineers decide which analytical method to use.

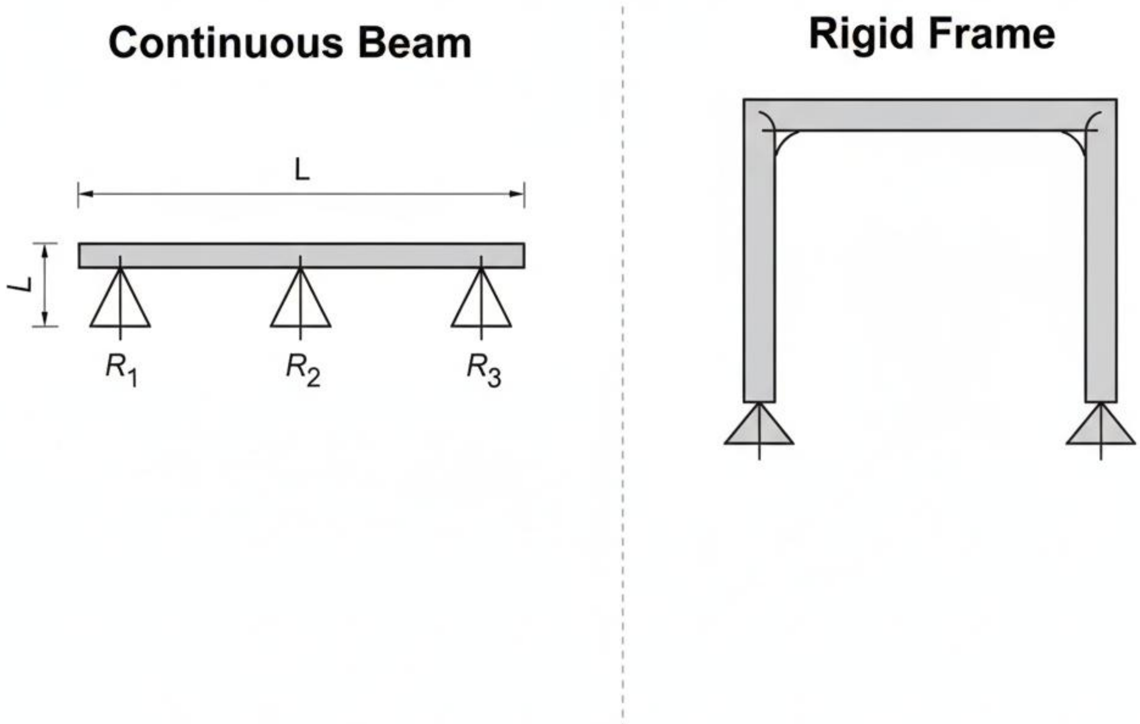


Fill in the blank: The degree of indeterminacy is the number of extra unknowns beyond the ________ equations.

5.2.3 Examples of indeterminate beams and frames

Indeterminate structures are common in practice. Continuous beams with more than two supports, fixed beams, and rigid frames are typical examples. These structures are preferred in many cases because they can resist larger loads and provide greater safety margins. However, their analysis is more complex and requires advanced methods such as the force method or displacement method.

Figure 5.4 Examples of indernadate structures: continuous beam and rigid frame



Fill in the blank: Continuous beams with more than two supports are examples of ________ structures.

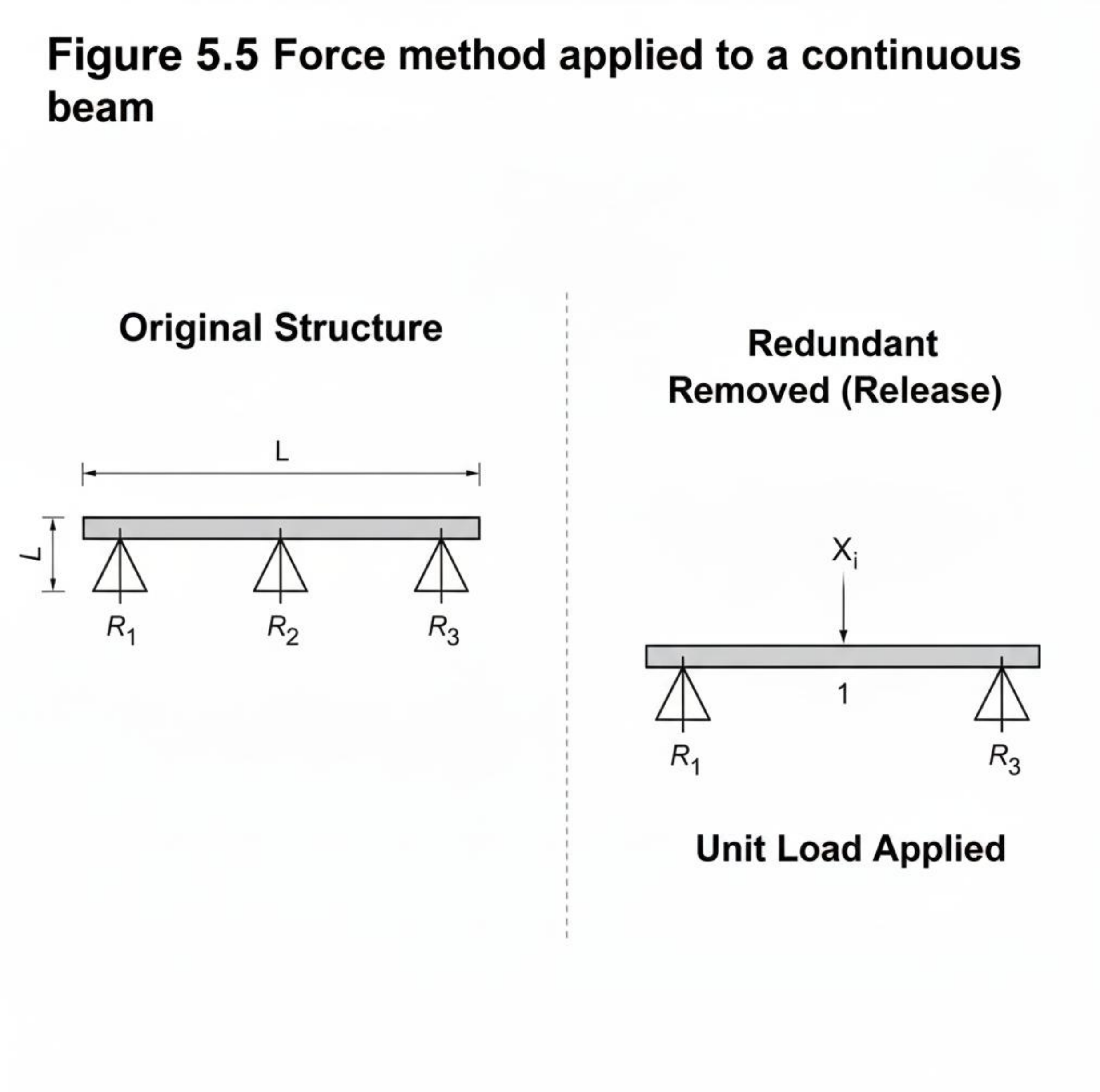
Pilot questions 5.2

1. What defines a statically indeterminate structure?
2. How is the degree of indeterminacy calculated?
3. Give one example of an indeterminate beam.
4. Why are indeterminate structures considered more resilient than determinate ones?
5. What additional condition is required to analyse indeterminate structures apart from equilibrium?

5.3 Basic Approaches To Indeterminate Analysis

5.3.1 Force method (flexibility method)

The **force method**, also known as the **flexibility method**, is one of the classical approaches to analysing statically indeterminate structures. It involves selecting redundant forces, removing them to make the structure determinate, and then applying compatibility conditions to solve for the redundants. This method is particularly effective for structures with a low degree of indeterminacy, as it reduces the problem to manageable equations.

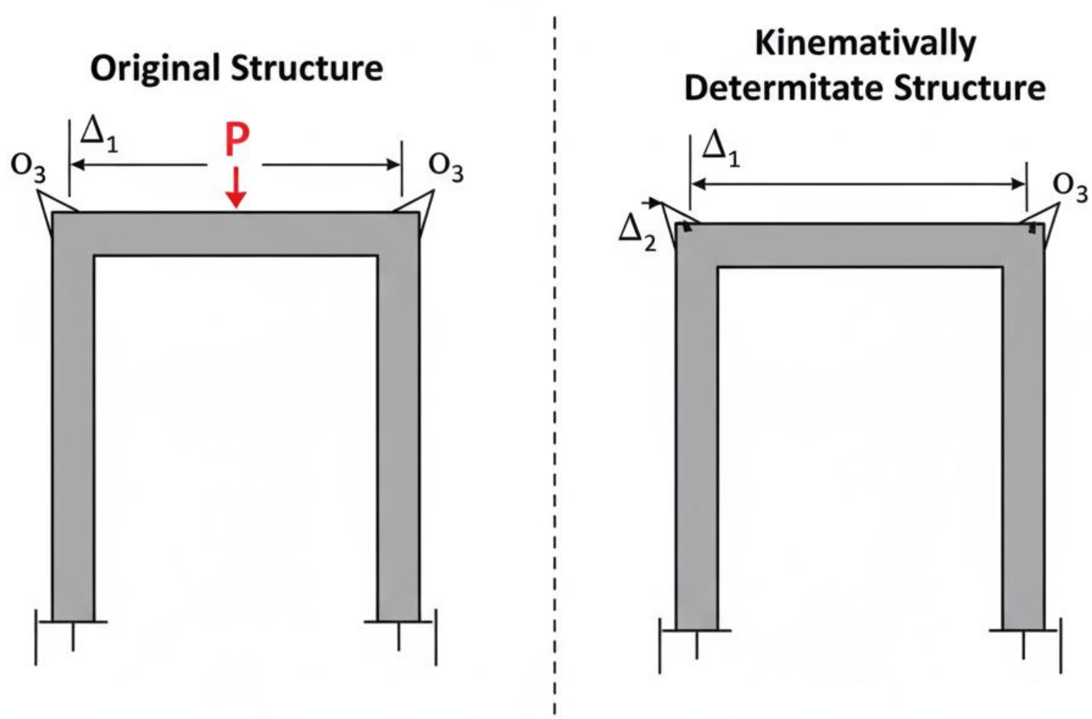


Fill in the blank: The force method involves removing ________ forces to make the structure determinate.

5.3.2 Displacement method (stiffness method)

The **displacement method**, also called the **stiffness method**, focuses on unknown displacements rather than forces. In this approach, the structure is modelled in terms of stiffness, and equations are formed relating applied loads to displacements. This method is particularly suitable for computer-based analysis and is widely used in modern structural engineering software.

Figure 5.6 Displacement method applied to a rigid frame



Fill in the blank: The displacement method focuses on unknown ________ rather than forces.

5.3.3 Practical relevance and limitations

Both the force method and displacement method are essential in structural analysis. The force method is straightforward for simple indeterminate structures, while the displacement method is more versatile and suited for complex systems. However, each has limitations: the force method becomes cumbersome for highly indeterminate structures, and the displacement method requires computational resources. Engineers often choose the method based on the complexity of the structure and the tools available.

Box 5.3 Comparison of force and displacement methods

Force Method	Displacement Medhon
Advantages	**Advanagens**
• Fewer equations if I < D • Can choose redundants strateigrly • Physical interpretation	• Systentic formulation • Easily programmble for computers • Easily programmble for • Handles all stable structures
Limitations	**Limitations**
• Cannot handle kinakemialy structures • Redudunt selection can be complex • Requires unit load method	• More equations if D > I • Less intutive physical interpetation • Fixed/pinned-end moments needed

Fill in the blank: The displacement method is more versatile but requires greater ________ resources.

Pilot questions 5.3

1. What is the main principle of the force method?
2. Why is the displacement method particularly suitable for computer-based analysis?
3. What type of structures are best analysed using the force method?
4. What is one limitation of the displacement method?
5. How do engineers decide which method to use for indeterminate analysis?

Series Summary

This Series has introduced you to deflection theorems and the fundamentals of statically indeterminate structures, both of which are essential for extending structural analysis beyond equilibrium. By focusing on displacement and redundancy, the Series has highlighted how engineers ensure safety, serviceability, and resilience in structures that cannot be solved using equilibrium equations alone.

We began with the unit load method and Castigliano's theorem, showing how they are applied to calculate displacements in beams and trusses. We then moved on to the definition and characteristics of indeterminate structures, examining degrees of indeterminacy and practical examples such as continuous beams and rigid frames. Finally, we explored the basic approaches to indeterminate analysis, including the force method and displacement method, and discussed their practical relevance and limitations. In line with the Series Learning Outcomes, you should now be able to state and explain deflection theorems, apply them to beams and trusses, define and classify indeterminate structures, determine their degree of indeterminacy, and demonstrate the use of both force and displacement methods in analysis.

Glossary of Terms

- **Castigliano's theorem**: A principle stating that the displacement at a point in a structure is found by differentiating the strain energy with respect to the applied load at that point.
- **Degree of indeterminacy**: The number of extra unknown forces or reactions in a structure beyond the available equilibrium equations.
- **Displacement method (stiffness method)**: An analytical approach that focuses on unknown displacements, using stiffness relationships to calculate forces and reactions.
- **Flexibility method (force method)**: A classical approach to indeterminate analysis that removes redundant forces to make a structure determinate, then applies compatibility conditions to solve for those forces.
- **Indeterminate structure**: A structure with more unknown forces or reactions than can be solved using equilibrium equations alone, requiring additional conditions for analysis.
- **Redundant forces**: Extra forces or reactions in a structure that make it indeterminate, usually removed temporarily in the force method to simplify analysis.
- **Strain energy**: The energy stored in a structure due to deformation under applied loads.
- **Unit load method**: A deflection theorem where a fictitious unit load is applied at the point of interest to calculate displacement.

Concept Check Questions 5

Objective questions

1. State the principle of the unit load method. (Linked to SLO 5.1)

2. Castigliano's theorem relates displacement to which structural property? (Linked to SLO 5.2)
3. Continuous beams with more than two supports are examples of what type of structure? (Linked to SLO 5.4)
4. The displacement method is also known as the ________ method. (Linked to SLO 5.8)
5. Which method becomes cumbersome for highly indeterminate structures? (Linked to SLO 5.7)

Short-answer questions

6. Explain how the unit load method is applied to calculate deflection in a beam. (Linked to SLO 5.3)
7. Define the degree of indeterminacy and give a simple example. (Linked to SLO 5.5)
8. Illustrate one example of an indeterminate frame commonly used in practice. (Linked to SLO 5.6)
9. Compare one advantage of the force method with one advantage of the displacement method. (Linked to SLO 5.9)

Long-answer questions

10. Analyse the importance of deflection theorems in structural mechanics, highlighting their applications in beams and trusses. (Linked to SLO 5.3)
11. Evaluate the practical relevance of indeterminate structures in engineering, discussing both their benefits and limitations. (Linked to SLO 5.9)

Answers to Concept Check Questions 5

Objective answers

1. It involves applying a fictitious unit load at the point of interest.
2. Strain energy.
3. Statically indeterminate structures.
4. Stiffness method.
5. Force method.

Short-answer answers

6. A unit load is applied at the point of interest, internal forces are calculated, and compatibility conditions are used to determine displacement.
7. It is the number of extra unknowns beyond equilibrium equations. For example, a beam with three supports has a degree of indeterminacy of one.
8. A rigid frame used in industrial buildings is a common example.
9. The force method is straightforward for low indeterminacy, while the displacement method is versatile for complex structures.

Long-answer answers

Question 10

- **Basic response**: Deflection theorems such as the unit load method and Castigliano's theorem allow engineers to calculate displacements in beams and trusses without solving complex differential equations. They are applied to ensure serviceability and safety.
- **Value-added response**: Beyond basic applications, deflection theorems form the foundation for advanced energy methods in structural analysis. They are integrated into computational tools and finite element methods, enabling engineers to analyse large-scale structures efficiently.

Question 11

- **Basic response**: Indeterminate structures provide redundancy, making them more resilient and capable of redistributing loads. However, they require more complex analysis and cannot be solved using equilibrium alone.
- **Value-added response**: In modern engineering, indeterminate structures are preferred for bridges, high-rise buildings, and industrial frames because they enhance safety margins and durability. Their analysis is facilitated by computer-based methods, but they demand careful consideration of material properties, load paths, and long-term behaviour such as creep and shrinkage.

Answers to Pilot Questions 5

Part 5.1 Deflection Theorems

- A fictitious unit load is applied at the point of interest.
- By differentiating strain energy with respect to the applied load.
- They allow calculation of displacements in beams and trusses.
- Strain energy.
- Calculating deflection in beams.

Part 5.2 Introduction to Statically Indeterminate Structures

- A structure with more unknown forces than equilibrium equations.
- By counting the extra unknowns beyond equilibrium equations.
- Continuous beam with three supports.
- They can redistribute loads and provide redundancy.
- Compatibility of deformations.

Part 5.3 Basic Approaches to Indeterminate Analysis

- Removing redundant forces to make the structure determinate.
- It models stiffness and relates loads to displacements.
- Structures with low degrees of indeterminacy.
- It requires computational resources.

- Based on complexity of the structure and available tools.

References and Attributions 5

This section compiles all references, suggested open educational resources (OERs), and illustration prompts used in **Series 5: Deflection Theorems and Indeterminate Structures**. References are presented in APA style for clarity and consistency.

General References

- Hibbeler, R. C. (2017). *Structural Analysis* (9th ed.). Pearson Education.
- Kassimali, A. (2018). *Structural Analysis* (5th ed.). Cengage Learning.
- McCormac, J. C., & Nelson, J. K. (2015). *Structural Analysis: A Classical and Matrix Approach*. Wiley.

Illustrations and Attributions

- *Figure 5.1 Application of unit load method to a beam*
  - AI-generated using prompt: "Generate a diagram of a simply supported beam with a unit load applied at mid-span."
  - Suggested OER search string: "OER unit load method beam diagram."
- *Figure 5.2 Castigliano's theorem applied to a loaded beam*
  - AI-generated using prompt: "Generate a diagram of a loaded beam with strain energy distribution to illustrate Castigliano's theorem."
  - Suggested OER search string: "OER Castigliano theorem beam diagram."
- *Box 5.1 Applications of deflection theorems*
  - AI-generated using prompt: "Create a text box summarising applications of deflection theorems in beams and trusses."
  - Suggested OER search string: "OER applications of deflection theorems summary."
- *Figure 5.3 Comparison of determinate and indeterminate beams*
  - AI-generated using prompt: "Generate a diagram comparing determinate and indeterminate beams with labelled supports and reactions."
  - Suggested OER search string: "OER determinate vs indeterminate beams diagram."
- *Box 5.2 Degrees of indeterminacy explained*
  - AI-generated using prompt: "Create a text box summarising degrees of indeterminacy with examples of beams and frames."
  - Suggested OER search string: "OER degrees of indeterminacy examples."
- *Figure 5.4 Examples of indeterminate structures: continuous beam and rigid frame*
  - AI-generated using prompt: "Generate a diagram showing a continuous beam with three supports and a rigid frame."
  - Suggested OER search string: "OER indeterminate structures examples diagram."
- *Figure 5.5 Force method applied to a continuous beam*

- AI-generated using prompt: “Generate a diagram of a continuous beam with a redundant support removed to illustrate the force method.”
- Suggested OER search string: “OER force method flexibility method beam diagram.”

- *Figure 5.6 Displacement method applied to a rigid frame*
  - AI-generated using prompt: “Generate a diagram of a rigid frame analysed using the displacement method with labelled displacements.”
  - Suggested OER search string: “OER displacement method stiffness method frame diagram.”
- *Box 5.3 Comparison of force and displacement methods*
  - AI-generated using prompt: “Create a text box comparing the advantages and limitations of the force method and displacement method.”
  - Suggested OER search string: “OER comparison force method displacement method.”