BED 122

BUSINESS MATHEMATICS

Course code: BED 122

Course title: Business Mathematics

Course credit load: (2 Units C: LH 30)

List of Series

SERIES 1 **Mathematics and Symbolic Logic**
SERIES 2 **Matrices, Determinants, and Vectors**
SERIES 3 **Complex Numbers and Coordinate Geometry**
SERIES 4 **Sequences, Series, and Calculus**
SERIES 5 **Optimization and Linear Programming**

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1 Mathematics and Symbolic Logic

Series Outline

- **Part 1: Foundations of Symbolic Logic**
  - Sub-Part 1.1: Basic concepts of logic
  - Sub-Part 1.2: Logical operators and truth tables
  - Sub-Part 1.3: Applications of symbolic logic in reasoning
- **Part 2: Deductive and Inductive Reasoning**
  - Sub-Part 2.1: Principles of deductive reasoning
  - Sub-Part 2.2: Principles of inductive reasoning
  - Sub-Part 2.3: Problem-solving approaches using reasoning
- **Part 3: Sets and Their Applications**
  - Sub-Part 3.1: Definition and types of sets
  - Sub-Part 3.2: Set operations and Venn diagrams
  - Sub-Part 3.3: Applications of sets in mathematics and business

This outline gives us **three main Parts**, which conforms to the guideline for a 2-credit course.

Introduction

Background
Mathematics is not only about numbers, it is also about reasoning and structure. Symbolic logic provides a systematic way of thinking, allowing us to analyse arguments and solve problems with clarity. In business mathematics, logical reasoning and the study of sets form the foundation for more advanced topics such as matrices, calculus, and optimisation. This Series will help you build these essential skills.

Series Aim
The aim of this Series is to equip you with the ability to apply symbolic logic and reasoning techniques, and to use sets effectively in mathematical and business contexts.

Series Synopsis
We shall commence this Series by exploring the foundations of symbolic logic, where you will learn about logical concepts, operators, and truth tables. Next, we will move on to deductive and inductive reasoning, examining how these approaches support problem-solving in mathematics and business. Finally, we will conclude with sets and their applications, focusing on definitions, operations, and practical uses in everyday problem-solving.

Series Learning Outcomes

Upon completing this Series, you should be able to:

- **SLO 1.1** define the basic concepts of symbolic logic,
- **SLO 1.2** explain the use of logical operators and construct truth tables,
- **SLO 1.3** apply deductive reasoning to solve structured problems,
- **SLO 1.4** analyse situations using inductive reasoning for generalisation, and
- **SLO 1.5** demonstrate how to perform set operations and interpret Venn diagrams in mathematical and business contexts.

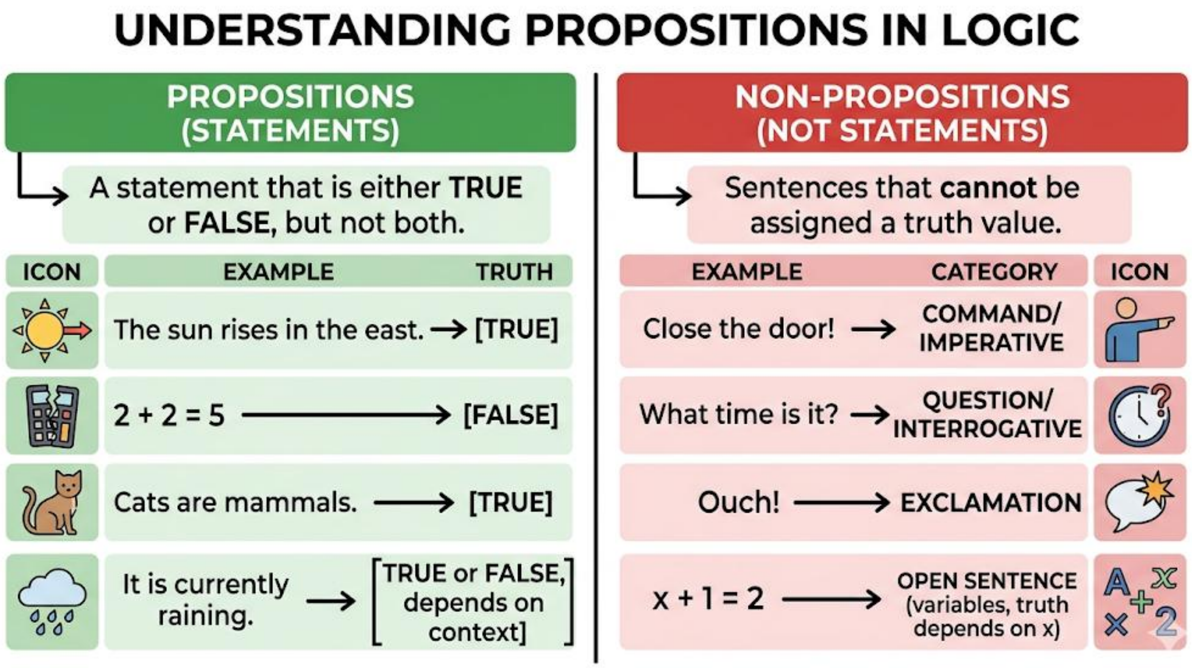
1.1 Foundations of Symbolic Logic

1.1.1 Basic concepts of logic

Logic is the study of reasoning. In mathematics, **symbolic logic** refers to the use of symbols to represent logical expressions and arguments. This allows us to analyse statements systematically and determine whether they are valid or invalid. A **proposition** is a declarative statement that can be either true or false, but not both. For example, “The sky is blue” is a proposition because it can be evaluated as true or false.

Fill in the blank: A statement that can be either true or false, but not both, is called a ________.

Diagram showing examples of propositions and non-propositions
Caption: Figure 1.1 Examples of propositions and non-propositions



1.1.2 Logical operators and truth tables

In symbolic logic, **logical operators** are symbols used to connect propositions and form compound statements. The most common operators are:

- **Negation (¬)**: reverses the truth value of a proposition.
- **Conjunction (∧)**: represents "and", true only if both propositions are true.
- **Disjunction (∨)**: represents "or", true if at least one proposition is true.
- **Implication (→)**: represents "if...then", true except when the first proposition is true and the second is false.
- **Biconditional (↔)**: represents "if and only if", true when both propositions have the same truth value.

A **truth table** is a tabular representation that shows all possible truth values of compound statements. It is a powerful tool for analysing logical expressions.

Fill in the blank: The operator that represents "if...then" is called ________.

Caption: Table 1.1 Truth table for conjunction ($p \wedge q$)

p	q	p∧q
T	T	**T**
T	F	**F**
F	T	**F**
F	F	**F**

1.1.3 Applications of symbolic logic in reasoning

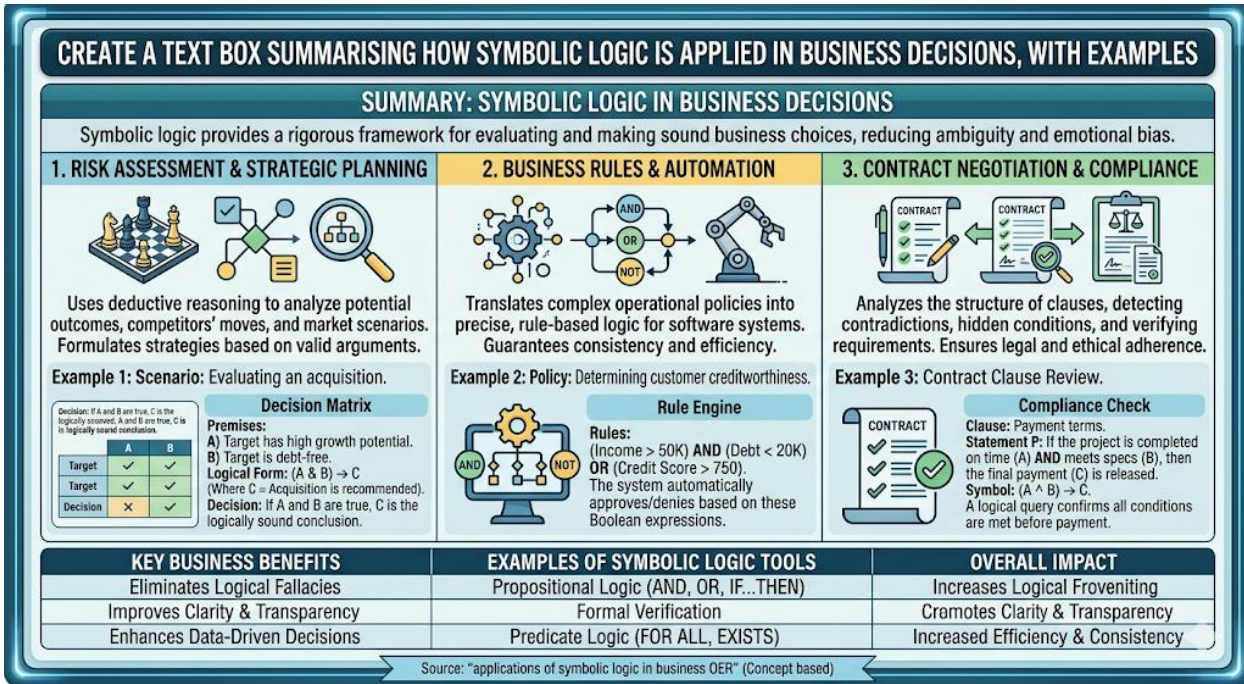
Symbolic logic is widely applied in mathematics, computer science, and business decision-making. For example, in programming, logical operators are used to control the flow of decisions. In business, logical reasoning helps managers evaluate scenarios and make consistent decisions.

Consider this example: A company decides to invest in a project if the market demand is high **and** production costs are low. Symbolic logic allows us to represent this decision as:

- p: Market demand is high
- q: Production costs are low
- Decision rule: $p \wedge q$

Fill in the blank: The decision rule "$p \wedge q$" will only be true if ________.

Caption: Box 1.1 Application of symbolic logic in business decision-making



Pilot questions 1.1

1. Which of the following is a proposition?
   a. Close the door
   b. The sun rises in the east
   c. What time is it
   d. Please help me

2. Which logical operator represents "and"?
   a. ∨
   b. ∧
   c. →
   d. ¬

3. In a truth table, the conjunction p ∧ q is true when:
   a. Both p and q are false
   b. At least one of p or q is true
   c. Both p and q are true
   d. p is true and q is false

4. Which operator reverses the truth value of a proposition?
   a. ∧
   b. ¬

c. →
d. ↔

5. A business decision rule expressed as p ∧ q will be true only when:
   a. p is true and q is false
   b. p is false and q is true
   c. both p and q are true
   d. both p and q are false

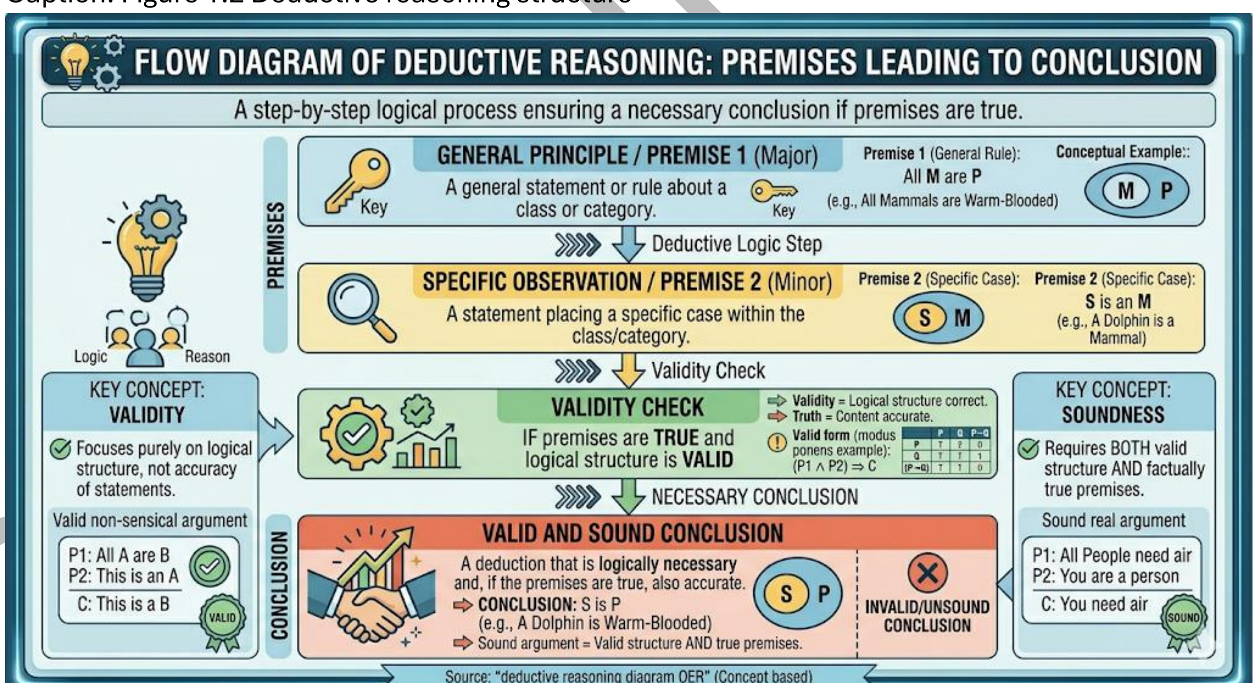
1.2 Deductive and Inductive Reasoning

1.2.1 Principles of deductive reasoning

Deductive reasoning is a logical process where conclusions are drawn from general statements or premises. If the premises are true, the conclusion must also be true. For example, if "All managers are employees" and "James is a manager", then we can deduce that "James is an employee". Deductive reasoning is often used in mathematics and business to ensure accuracy and reliability.

Fill in the blank: If all managers are employees and James is a manager, then James is an ________.

Diagram showing deductive reasoning with premises and conclusion
Caption: Figure 1.2 Deductive reasoning structure



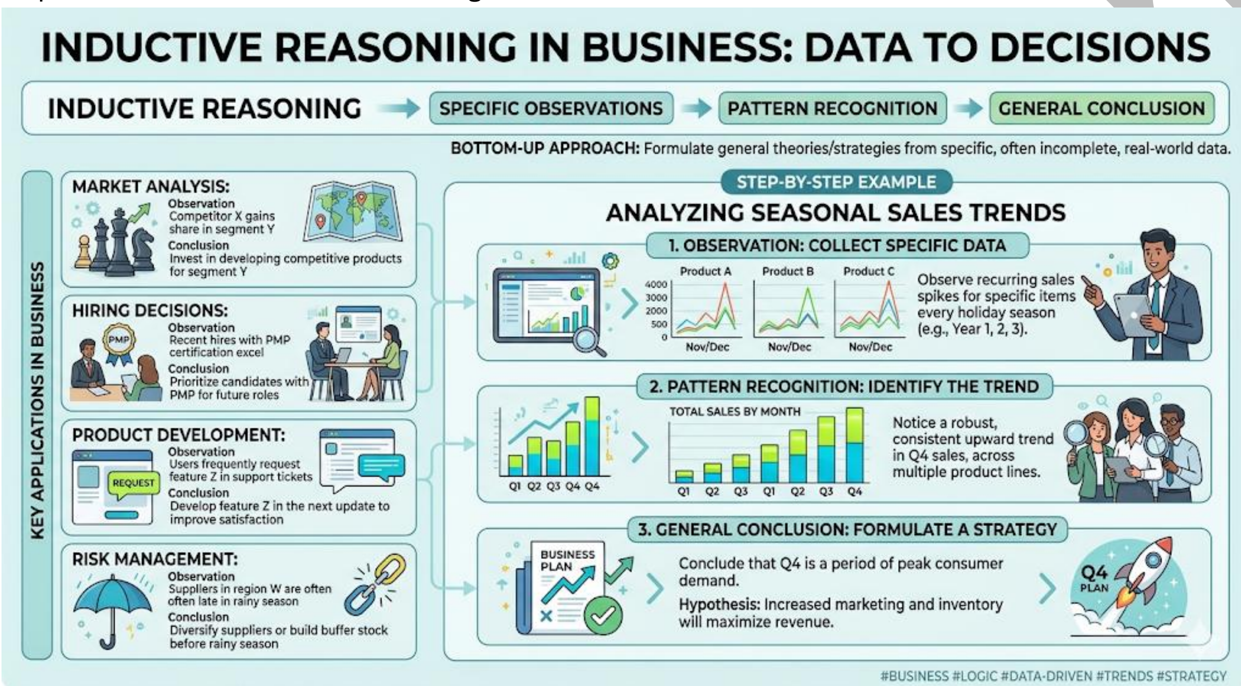
1.2.2 Principles of inductive reasoning

Inductive reasoning involves drawing general conclusions from specific observations. Unlike deduction, inductive reasoning does not guarantee certainty, but it provides probable conclusions.

For example, if a business notices that sales increase every December, it may induce that December is a peak season.

Fill in the blank: Inductive reasoning moves from ________ observations to general conclusions.

Image showing inductive reasoning with examples from business trends
Caption: Plate 1.1 Inductive reasoning in business



1.2.3 Problem-solving approaches using reasoning

Both deductive and inductive reasoning are essential in problem-solving. Deduction ensures logical certainty, while induction helps in predicting patterns and making informed decisions. In business mathematics, combining both approaches allows managers to evaluate risks and opportunities effectively.

Fill in the blank: Deductive reasoning ensures ________, while inductive reasoning provides probable conclusions.

Caption: Box 1.2 Comparing deductive and inductive reasoning

Feature	Deductive Reasoning	Inductive Reasoning
Direction	**Top-Down:** Starts with a general theory and moves toward a specific conclusion.	**Bottom-Up:** Starts with specific observations and moves toward a broad generalization.
Definition	The process of reaching a logically certain conclusion from one or more general statements (premises).	The process of making broad generalizations based on specific observations or patterns.
Certainty	If the premises are true, the conclusion **must** be true (Valid/Sound).	The conclusion is **probabilistic**; it is likely true but not guaranteed (Strong/Cogent).
Goal	To **prove** or verify a specific point based on established rules or laws.	To **predict** or develop a new theory based on observed data.
Example	**Premise A:** All human beings require oxygen to live. **Premise B:** John is a human being. **Conclusion:** Therefore, John requires oxygen to live.	**Observation A:** The sun has risen in the east every morning of my life. **Observation B:** Meteorological records show this is consistent. **Conclusion:** Therefore, the sun will rise in the east tomorrow.

Pilot questions 1.2

1. Deductive reasoning moves from:
   a. Specific to general
   b. General to specific
   c. Probable to certain
   d. Observations to predictions
2. Inductive reasoning provides:
   a. Absolute certainty
   b. Probable conclusions

c. Mathematical proofs
d. Logical contradictions

3. Which reasoning method is used when predicting seasonal sales trends?
   a. Deductive reasoning
   b. Inductive reasoning
   c. Both deductive and inductive
   d. Neither

4. If all accountants are employees and Sarah is an accountant, what can be deduced?
   a. Sarah is not an employee
   b. Sarah is an employee
   c. Sarah is a manager
   d. Sarah is a customer

5. Which reasoning method ensures logical certainty?
   a. Inductive reasoning
   b. Deductive reasoning
   c. Both
   d. None

1.3 Sets and Their Applications

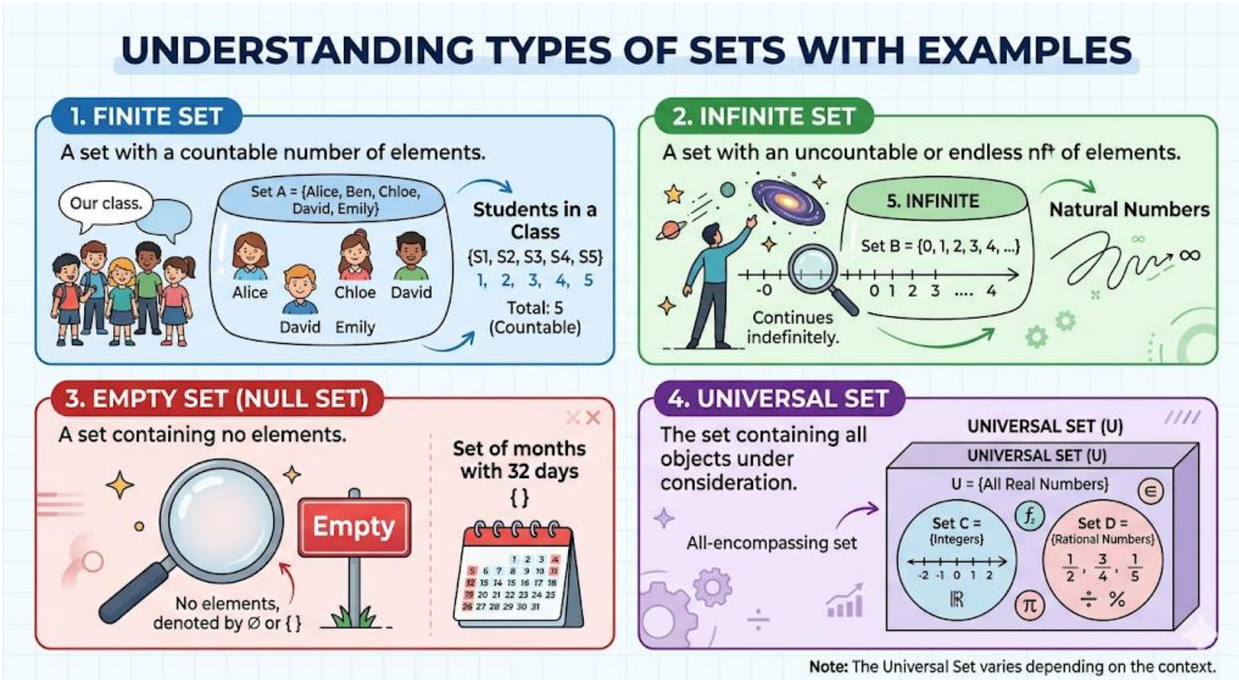
1.3.1 Definition and types of sets

A **set** is a collection of distinct objects, called elements, considered as a whole. Sets are usually denoted by curly brackets, for example, A = {2, 4, 6}. Types of sets include finite sets, infinite sets, empty sets, and universal sets.

Fill in the blank: A collection of distinct objects considered as a whole is called a ________.

Diagram showing different types of sets

Caption: Figure 1.3 Types of sets



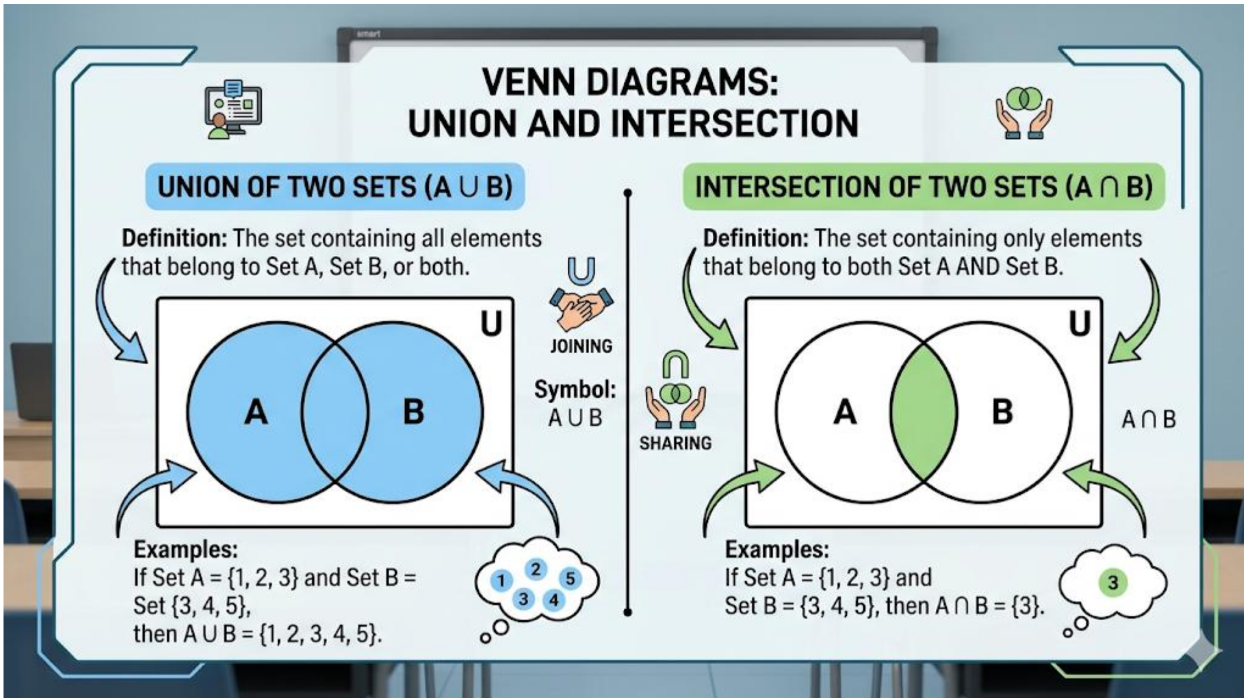
1.3.2 Set operations and Venn diagrams

Set operations include union, intersection, difference, and complement. These operations help in combining or comparing sets. A **Venn diagram** is a graphical representation of sets using overlapping circles to show relationships.

Fill in the blank: The operation that combines all elements from two sets is called ________.

Diagram of a Venn diagram showing union and intersection

Caption: Figure 1.4 Venn diagram of union and intersection



1.3.3 Applications of sets in mathematics and business

Sets are applied in mathematics to organise data and solve problems. In business, sets can represent customer groups, product categories, or market segments. For example, a company may define one set as customers who purchased product A and another set as customers who purchased product B. Analysing the intersection helps identify customers who purchased both products.

Fill in the blank: In business, sets can represent customer groups, product categories, or ________.

Caption: Box 1.3 Applications of sets in business

Application	Description	Business Example
Market Segmentation	Categorizing a broad consumer base into sub-groups (sets) based on shared characteristics.	A clothing retailer defines **Set A** as "Customers over 30" and **Set B** as "High-income earners" to target luxury professional wear.
Database Management	Using intersections and unions to filter customer or inventory data in CRM systems.	A manager queries the intersection of **Set "Past Buyers"** and **Set "Newsletter Subscribers"** to send a specific discount code.

Application	Description	Business Example
Inventory Control	Organizing products into distinct sets to manage stock levels, expiration dates, or categories.	A grocery store manages a set of **"Perishable Goods"** to prioritize rapid turnover and reduce waste.
Human Resources	Matching employee skill sets with job requirements during the recruitment process.	An IT firm looks for the intersection of candidates in the **"Java Programming" set** and the **"Project Management" set**.
Risk Management	Identifying overlapping risk factors that could affect different departments or projects.	A bank identifies the union of **"Credit Risk"** and **"Market Volatility"** sets to calculate total potential exposure.

Pilot questions 1.3

1. Which of the following is a set?
   a. {1, 2, 3}
   b. {apple, banana, orange}
   c. {students in a class}
   d. All of the above
2. The union of two sets includes:
   a. Only common elements
   b. All elements from both sets
   c. Only unique elements
   d. None of the above
3. Which diagram is used to represent sets graphically?
   a. Flowchart
   b. Venn diagram
   c. Bar chart
   d. Pie chart
4. The intersection of two sets includes:
   a. Elements present in both sets
   b. Elements present in only one set
   c. Elements excluded from both sets
   d. None of the above

5. In business, analysing the intersection of two customer sets helps identify:
   a. Customers who purchased only one product
   b. Customers who purchased both products
   c. Customers who purchased neither product
   d. Customers who purchased all products

Series Summary

This Series has introduced you to the foundations of symbolic logic and its importance in mathematical and business reasoning. We began by examining the basic concepts of logic, including propositions, logical operators, and truth tables, which provide a structured way of analysing statements. We then explored deductive and inductive reasoning, highlighting how deduction ensures certainty while induction supports generalisation and prediction. Finally, we studied sets, their operations, and applications, with Venn diagrams providing a visual tool for understanding relationships.

By completing this Series, you should now be able to define and explain symbolic logic, apply deductive and inductive reasoning in problem-solving, and demonstrate how sets and their operations can be used in mathematics and business contexts. These skills directly align with the Series Learning Outcomes and form a strong foundation for the more advanced topics that follow in the course.

Glossary of Terms

- **Biconditional (↔):** A logical operator that is true when both propositions have the same truth value.
- **Conjunction (∧):** A logical operator meaning "and", true only if both propositions are true.
- **Deductive reasoning:** A logical process that moves from general premises to specific conclusions with certainty.
- **Disjunction (∨):** A logical operator meaning "or", true if at least one proposition is true.
- **Inductive reasoning:** A logical process that moves from specific observations to general conclusions, providing probability rather than certainty.
- **Logical operator:** A symbol used to connect propositions and form compound statements.
- **Negation (¬):** A logical operator that reverses the truth value of a proposition.
- **Proposition:** A declarative statement that can be either true or false, but not both.
- **Set:** A collection of distinct objects considered as a whole.

- **Truth table:** A tabular representation showing all possible truth values of compound statements.
- **Venn diagram:** A graphical representation of sets using overlapping circles to show relationships.

Concept Check Questions 1

Objective questions

1. Which logical operator represents "and"? (Linked to SLO 1.2)
2. A proposition is defined as: (Linked to SLO 1.1)
3. Which reasoning method ensures logical certainty? (Linked to SLO 1.3)

Short-answer questions

4. Explain the difference between deductive and inductive reasoning with one example each. (Linked to SLO 1.3 and SLO 1.4)
5. Define a set and give two examples relevant to business. (Linked to SLO 1.5)

Long-answer questions

6. Analyse how symbolic logic can be applied in business decision-making, using an example of a company evaluating investment options. (Linked to SLO 1.2 and SLO 1.3)
7. Demonstrate how Venn diagrams can be used to represent customer groups in a business scenario, and evaluate the usefulness of this approach. (Linked to SLO 1.5)

Answers to Concept Check Questions 1

Objective questions

1. $\wedge$
2. A declarative statement that can be either true or false.
3. Deductive reasoning

Short-answer questions

4. Deductive reasoning moves from general premises to specific conclusions, for example, "All managers are employees, James is a manager, therefore James is an employee." Inductive reasoning moves from specific observations to general conclusions, for example, "Sales increase every December, therefore December is a peak season."

5. A set is a collection of distinct objects considered as a whole. In business, examples include {customers who purchased product A} and {customers who purchased product B}.

Long-answer questions

6. *Basic response:* Symbolic logic helps businesses make consistent decisions. For example, a company may invest in a project if market demand is high and production costs are low, represented as $p \wedge q$.
   Value-added response: Beyond this, symbolic logic can be extended to complex decision-making involving multiple variables, such as market trends, competitor actions, and resource availability, allowing managers to evaluate scenarios systematically.

7. *Basic response:* Venn diagrams can represent customer groups, such as those who purchased product A, product B, or both. This helps identify overlaps and unique segments.
   Value-added response: Venn diagrams also support strategic marketing decisions, enabling businesses to target overlapping customers with bundled offers and unique segments with tailored promotions.

Answers to Pilot Questions [Series 1]

Part 1: Foundations of Symbolic Logic

1. b
2. b
3. c
4. b
5. c

Part 2: Deductive and Inductive Reasoning

1. b
2. b
3. b
4. b
5. b

Part 3: Sets and Their Applications

1. d
2. b

3. b
4. a
5. b

References and Attributions [Series 1]

- Suggested illustration sources:
    - Figure 1.1 Examples of propositions and non-propositions: “examples of propositions in logic diagram OER”
    - Table 1.1 Truth table for conjunction: “truth table for conjunction OER”
    - Figure 1.2 Deductive reasoning structure: “deductive reasoning diagram OER”
    - Plate 1.1 Inductive reasoning in business: “inductive reasoning examples business OER”
    - Box 1.2 Comparing deductive and inductive reasoning: “deductive vs inductive reasoning comparison OER”
    - Figure 1.3 Types of sets: “types of sets mathematics diagram OER”
    - Figure 1.4 Venn diagram of union and intersection: “Venn diagram union intersection OER”
    - Box 1.3 Applications of sets in business: “applications of sets in business OER”
- AI prompt suggestions were provided for each illustration to support future generation using AI tools.
- References:
    - Open Educational Resources (OER) in mathematics and logic.
    - Standard texts in business mathematics and symbolic logic.

Series 2: Matrices, Determinants, and Vectors

- **Part 2.1: Introduction to Matrices**
    - Sub-Part 2.1.1: Definition and types of matrices
    - Sub-Part 2.1.2: Matrix operations (addition, subtraction, multiplication)

- Sub-Part 2.1.3: Special matrices and their properties

- **Part 2.2: Determinants and Their Applications**
    - Sub-Part 2.2.1: Definition and calculation of determinants
    - Sub-Part 2.2.2: Properties of determinants
    - Sub-Part 2.2.3: Applications of determinants in solving equations
- **Part 2.3: Vectors and Their Properties**
    - Sub-Part 2.3.1: Definition and representation of vectors
    - Sub-Part 2.3.2: Vector operations (addition, scalar multiplication, dot product, cross product)
    - Sub-Part 2.3.3: Applications of vectors in mathematics and business

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Introduction

Background
In mathematics and business applications, matrices, determinants, and vectors provide powerful tools for organising data, solving equations, and modelling real-world problems. From analysing financial systems to representing physical quantities, these concepts form the backbone of many advanced techniques in business mathematics.

Series Aim
The aim of this Series is to introduce you to matrices, determinants, and vectors, and to equip you with the skills needed to perform operations, analyse properties, and apply these concepts in practical problem-solving.

Series Synopsis
We shall commence this Series by exploring matrices, where you will learn their definitions, types, and operations. Next, we will move on to determinants, examining how they are calculated, their properties, and their applications in solving systems of equations. Finally, we will conclude with vectors, focusing on their representation, operations, and applications in mathematics and business contexts.

Series Learning Outcomes

Upon completing this Series, you should be able to:

- **SLO 2.1** define matrices and explain their types,
- **SLO 2.2** demonstrate how to perform basic matrix operations,

- **SLO 2.3** calculate determinants and explain their properties,
- **SLO 2.4** apply determinants to solve systems of equations,
- **SLO 2.5** define vectors and represent them graphically,
- **SLO 2.6** perform vector operations including dot and cross products, and
- **SLO 2.7** analyse applications of vectors in mathematics and business contexts.

2.1 Introduction to Matrices

2.1.1 Definition and types of matrices

A **matrix** is a rectangular arrangement of numbers, symbols, or expressions, organised in rows and columns. Each number in a matrix is called an **element**, and the size of a matrix is defined by the number of rows and columns it contains. For example, a matrix with 2 rows and 3 columns is referred to as a 2 × 3 matrix.

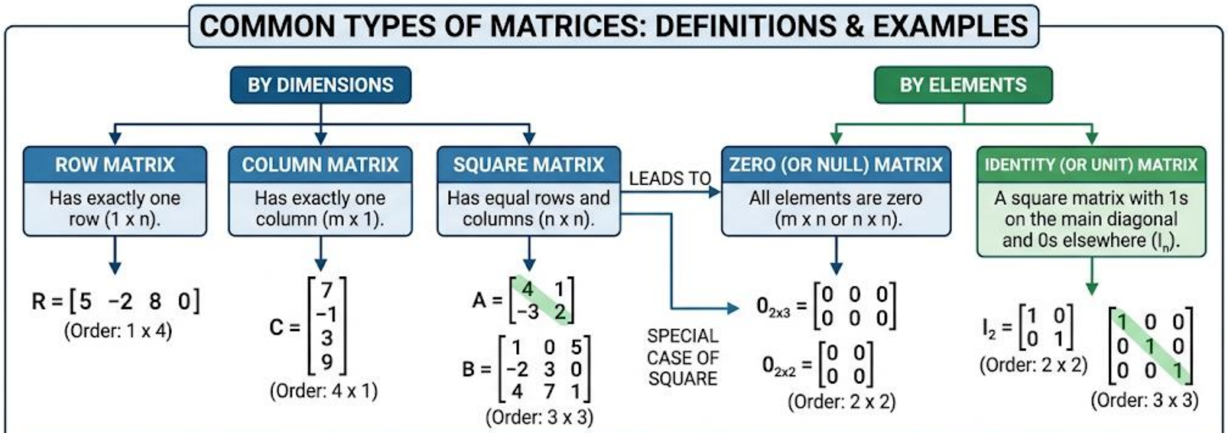
Types of matrices include:

- **Row matrix:** A matrix with only one row.
- **Column matrix:** A matrix with only one column.
- **Square matrix:** A matrix with the same number of rows and columns.
- **Zero matrix:** A matrix in which all elements are zero.
- **Identity matrix:** A square matrix with ones on the main diagonal and zeros elsewhere.

Fill in the blank: A matrix with the same number of rows and columns is called a ________.

Diagram showing different types of matrices with examples

Caption: Figure 2.1 Types of matrices



MATRIX TYPE	DESCRIPTION	EXAMPLE	NOTES
ROW	1 ROW	[−1 0 4]	Also "Row Vector"
COLUMN	1 COLUMN	[2] [5] [−3]	Also "Column Vector"
SQUARE	ROWS = COLUMNS	[3 1] [9 2]	Main diagonal exists
ZERO	ALL ELEMENTS = 0	[0 0] [0 0]	Can be any size
IDENTITY	SQUARE, DIAGONAL = 1, OTHERS = 0	[1 0 0] [0 1 0] [0 0 1]	Acts like number 1

2.1.2 Matrix operations (addition, subtraction, multiplication)

Matrices can be manipulated using various operations. **Matrix addition** and **subtraction** are performed by adding or subtracting corresponding elements of matrices of the same size. **Matrix multiplication** is more complex:

- Scalar multiplication involves multiplying every element of a matrix by a constant.
- Matrix multiplication involves multiplying rows of the first matrix by columns of the second matrix, provided the number of columns in the first equals the number of rows in the second.

Fill in the blank: Matrix multiplication is possible only when the number of columns in the first matrix equals the number of ________ in the second matrix.

Caption: Table 2.1 Example of matrix multiplication

$$\begin{array}{ccccc} \text{Matrix A} & & \text{Matrix B} & & \text{Result C} \\ \begin{bmatrix} a & b \\ c & d \end{bmatrix} & \times & \begin{bmatrix} e & f \\ g & h \end{bmatrix} & = & \begin{bmatrix} c11 & c12 \\ c21 & c22 \end{bmatrix} \end{array}$$

Step-by-Step Breakdown:

```
Step 1 (Top Left):  Row 1 of A * Col 1 of B  =>  (a * e) + (b * g)
```

```
Step 2 (Top Right): Row 1 of A * Col 2 of B  =>  (a * f) + (b * h)
```

```
Step 3 (Bot Left):  Row 2 of A * Col 1 of B  =>  (c * e) + (d * g)
```

```
Step 4 (Bot Right): Row 2 of A * Col 2 of B  =>  (c * f) + (d * h)
```

2.1.3 Special matrices and their properties

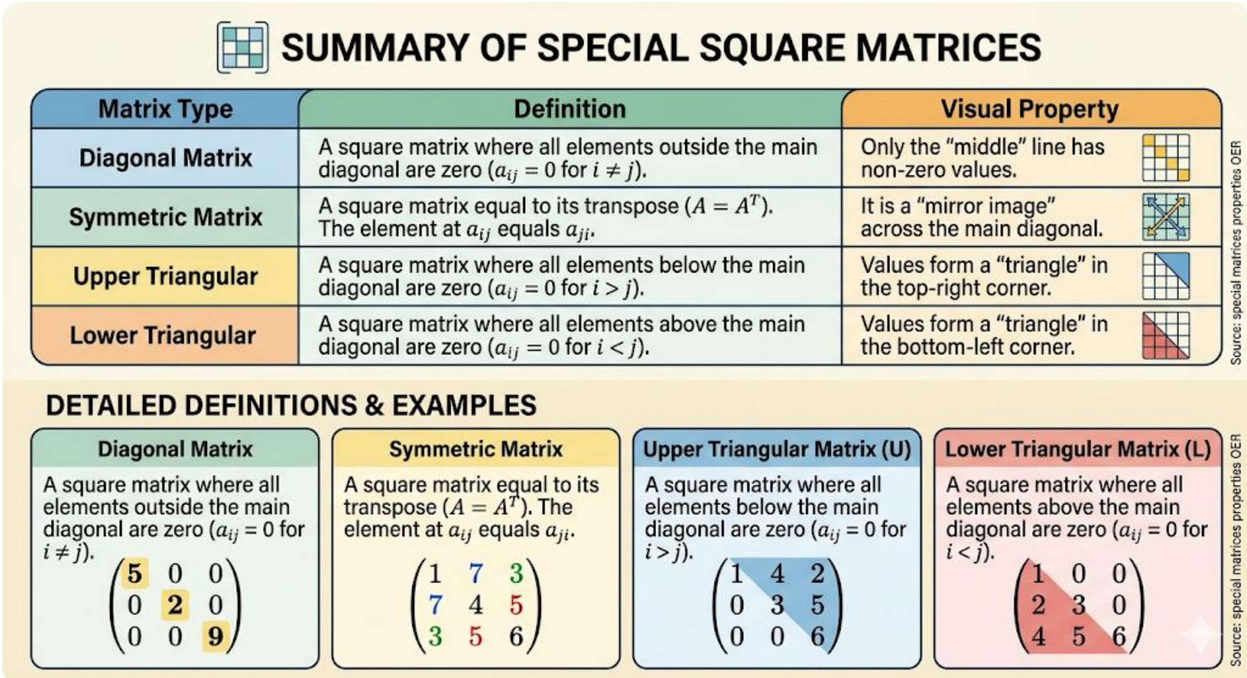
Certain matrices have unique properties that make them useful in mathematics and business applications. Examples include:

- **Diagonal matrix:** A square matrix where all non-diagonal elements are zero.
- **Symmetric matrix:** A square matrix that is equal to its transpose.
- **Upper triangular matrix:** A square matrix where all elements below the main diagonal are zero.
- **Lower triangular matrix:** A square matrix where all elements above the main diagonal are zero.

These special matrices are often used in solving linear equations, simplifying computations, and modelling systems.

Fill in the blank: A square matrix that is equal to its transpose is called a ________ matrix.

Caption: Box 2.1 Properties of special matrices



SUMMARY OF SPECIAL SQUARE MATRICES

Matrix Type	Definition	Visual Property
Diagonal Matrix	A square matrix where all elements outside the main diagonal are zero ($a_{ij} = 0$ for $i \neq j$).	Only the "middle" line has non-zero values.
Symmetric Matrix	A square matrix equal to its transpose ($A = A^T$). The element at a_{ij} equals a_{ji}.	It is a "mirror image" across the main diagonal.
Upper Triangular	A square matrix where all elements below the main diagonal are zero ($a_{ij} = 0$ for $i > j$).	Values form a "triangle" in the top-right corner.
Lower Triangular	A square matrix where all elements above the main diagonal are zero ($a_{ij} = 0$ for $i < j$).	Values form a "triangle" in the bottom-left corner.

Source: special matrices properties OER

DETAILED DEFINITIONS & EXAMPLES

Diagonal Matrix

A square matrix where all elements outside the main diagonal are zero ($a_{ij} = 0$ for $i \neq j$).

$$\begin{pmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 9 \end{pmatrix}$$

Symmetric Matrix

A square matrix equal to its transpose ($A = A^T$). The element at a_{ij} equals a_{ji}.

$$\begin{pmatrix} 1 & 7 & 3 \\ 7 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$$

Upper Triangular Matrix (U)

A square matrix where all elements below the main diagonal are zero ($a_{ij} = 0$ for $i > j$).

$$\begin{pmatrix} 1 & 4 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 6 \end{pmatrix}$$

Lower Triangular Matrix (L)

A square matrix where all elements above the main diagonal are zero ($a_{ij} = 0$ for $i < j$).

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{pmatrix}$$

Source: special matrices properties OER

Pilot questions 2.1

1. Which of the following is a square matrix?
   a. A matrix with 2 rows and 3 columns
   b. A matrix with 3 rows and 3 columns
   c. A matrix with 1 row and 4 columns
   d. A matrix with 4 rows and 1 column

2. Which type of matrix has ones on the main diagonal and zeros elsewhere?
   a. Zero matrix
   b. Identity matrix
   c. Diagonal matrix
   d. Symmetric matrix

3. Matrix addition is possible only when:
   a. The matrices are square
   b. The matrices have the same size
   c. The matrices are symmetric
   d. The matrices are diagonal

4. Which matrix is equal to its transpose?
   a. Symmetric matrix
   b. Diagonal matrix
   c. Upper triangular matrix
   d. Zero matrix

5. Matrix multiplication is possible only when:
    a. The number of rows in the first equals the number of rows in the second
    b. The number of columns in the first equals the number of rows in the second
    c. The number of columns in both matrices are equal
    d. The matrices are square

2.2 Determinants and Their Applications

2.2.1 Definition and calculation of determinants

A **determinant** is a scalar value that can be computed from a square matrix. It provides important information about the matrix, such as whether it is invertible. For a 2 × 2 matrix:

[\text{det} \begin{bmatrix} a & b \ c & d \end{bmatrix} = ad - bc]

For larger matrices, determinants are calculated using expansion methods such as cofactor expansion.

Fill in the blank: The determinant of a 2 × 2 matrix (\begin{bmatrix} a & b \ c & d \end{bmatrix}) is equal to ________.

Caption: Table 2.2 Step-by-step calculation of a 3 × 3 determinant

3x3 Determinant Calculation: Cofactor Expansion along Row 1

Step	Actioe Matrix & Choose Row	Formula & Matrix Operations	Calculation & Result
Step 1	$A = \begin{bmatrix} 3 & 1 & 2 \\ 0 & 4 & 5 \\ -2 & -1 & 0 \end{bmatrix}$ Start with 3x3 matrix and choose Row 1 for expansion.	$\lvert A\rvert = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$	
Step 2	Calculate C_{11} Remove Row 1, Col 1. Calculate Minor M_{11} and Cofactor C_{11}.	$A = \begin{bmatrix} \boxed{3} & 1 & 2 \\ 0 & 4 & 5 \\ -2 & -1 & 0 \end{bmatrix} \Rightarrow \lvert M_{11}\rvert = \begin{vmatrix} 4 & 5 \\ -1 & 0 \end{vmatrix}$	$C_{11} = (-1)^{1+1} \cdot ((4 \times 0) - (5 \times -1))$ $= 1 \cdot 5 = 5$
Step 3	Calculate C_{12} Remove Row 1, Col 2. Calculate Minor M_{12} and Cofactor C_{12}.	$A = \begin{bmatrix} 3 & \boxed{1} & 2 \\ 0 & 4 & 5 \\ -2 & -1 & 0 \end{bmatrix} \Rightarrow \lvert M_{12}\rvert = \begin{vmatrix} 0 & 5 \\ -2 & 0 \end{vmatrix}$	$C_{12} = (-1)^{1+2} \cdot ((0 \times 0) - (5 \times -2))$ $= -1 \cdot 10 = -10$
Step 4	Calculate C_{13} Remove Row 1, Col 3. Calculate Minor M_{13} and Cofactor C_{13}.	$A = \begin{bmatrix} 3 & 1 & \boxed{2} \\ 0 & 4 & 5 \\ -2 & -1 & 0 \end{bmatrix} \Rightarrow \lvert M_{13}\rvert = \begin{vmatrix} 0 & 4 \\ -2 & -1 \end{vmatrix}$	$C_{13} = (-1)^{1+3} \cdot ((0 \times -1) - (4 \times -2))$ $= 1 \cdot 8 = 8$
Step 5	Sinal Sum Sum the products of each element in Row 1 and its cofactor.	$\lvert A\rvert = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$	$\lvert A\rvert = (3 \times 5) + (1 \times -10) + (2 \times 8)$ $= 15 - 10 + 16 = 21$

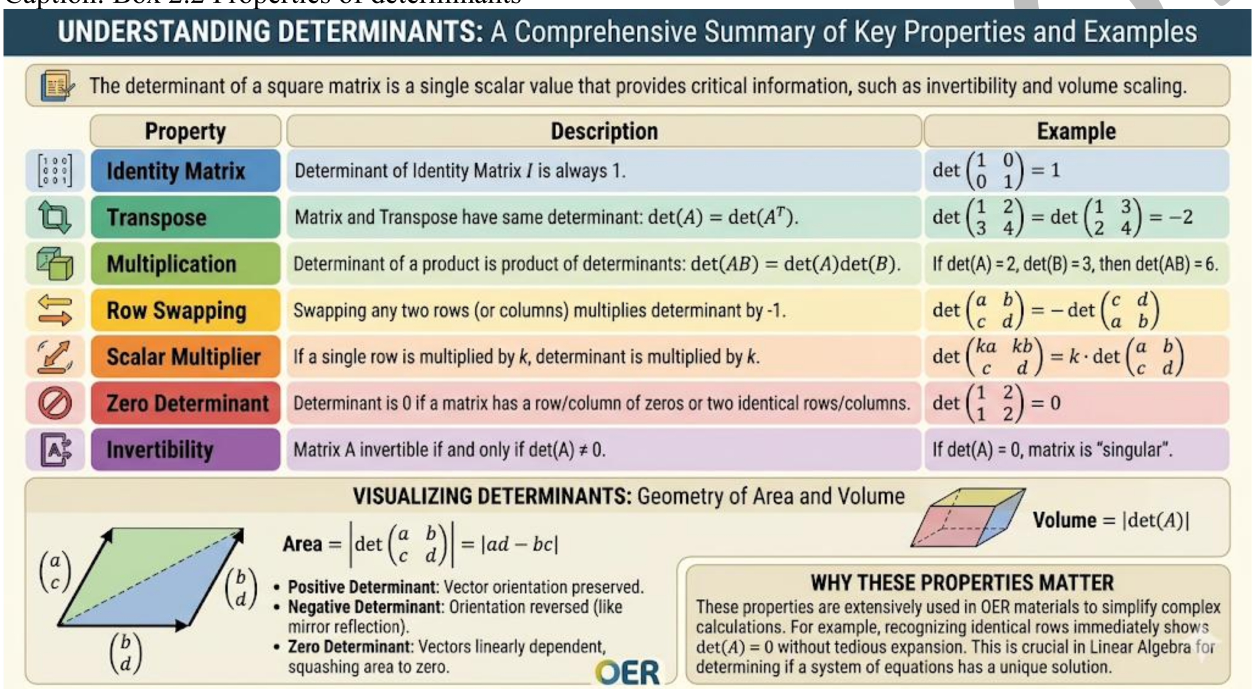
2.2.2 Properties of determinants

Determinants have several useful properties:

- The determinant of the identity matrix is 1.
- If two rows (or columns) of a matrix are identical, its determinant is 0.
- Swapping two rows (or columns) changes the sign of the determinant.
- Multiplying a row (or column) by a scalar multiplies the determinant by that scalar.

Fill in the blank: If two rows of a matrix are identical, its determinant is ________.

Caption: Box 2.2 Properties of determinants



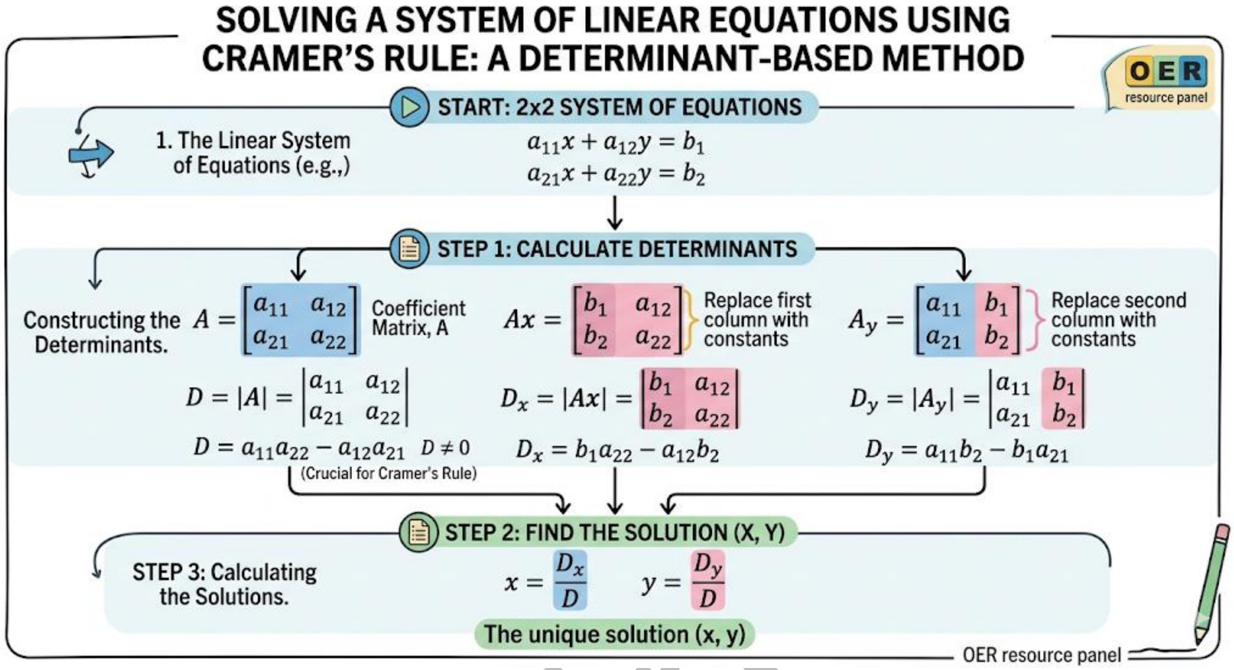
2.2.3 Applications of determinants in solving equations

Determinants are widely used in solving systems of linear equations. One important application is **Cramer's Rule**, which uses determinants to find solutions to simultaneous equations. Determinants also help in checking whether a matrix is invertible, which is crucial in mathematical modelling and business computations.

Fill in the blank: A matrix is invertible if and only if its determinant is ________.

Diagram showing application of determinants in solving a system of equations using Cramer's Rule

Caption: Figure 2.2 Application of determinants in solving equations



Pilot questions 2.2

1. The determinant of a 2 × 2 matrix (\begin{bmatrix} 2 & 3 \ 4 & 5 \end{bmatrix}) is:
   a. 2
   b. -2
   c. -2
   d. 10
2. The determinant of the identity matrix is:
   a. 0
   b. 1
   c. -1
   d. 2
3. If two rows of a matrix are identical, the determinant is:
   a. 1
   b. 0
   c. -1
   d. Undefined
4. Swapping two rows of a matrix changes the determinant by:
   a. Doubling it
   b. Halving it
   c. Changing its sign
   d. Leaving it unchanged
5. A matrix is invertible if its determinant is:
   a. 0
   b. Non-zero
   c. Negative
   d. Positive

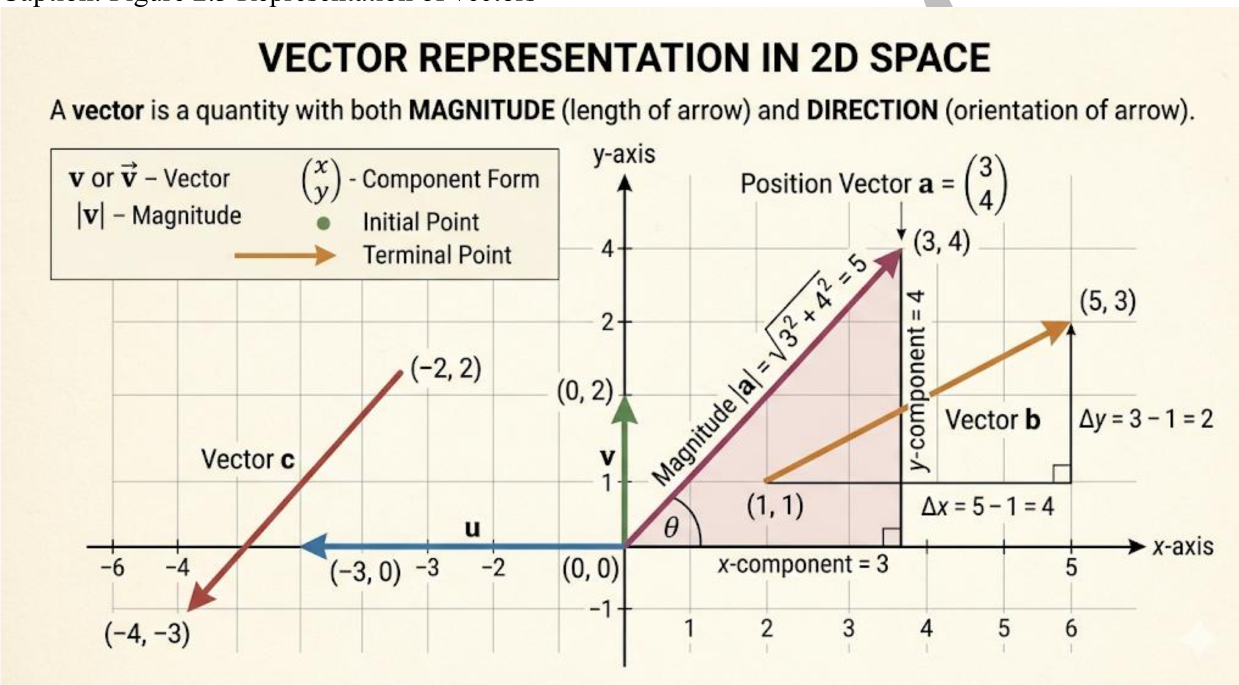
2.3 Vectors and Their Properties

2.3.1 Definition and representation of vectors

A **vector** is a quantity that has both magnitude and direction. Vectors are often represented as arrows in diagrams or as ordered pairs in mathematics. For example, a vector in two dimensions can be written as (\mathbf{v} = (x, y)).

Fill in the blank: A vector is defined as a quantity that has both ________ and direction.

Diagram showing graphical representation of vectors in two dimensions
Caption: Figure 2.3 Representation of vectors



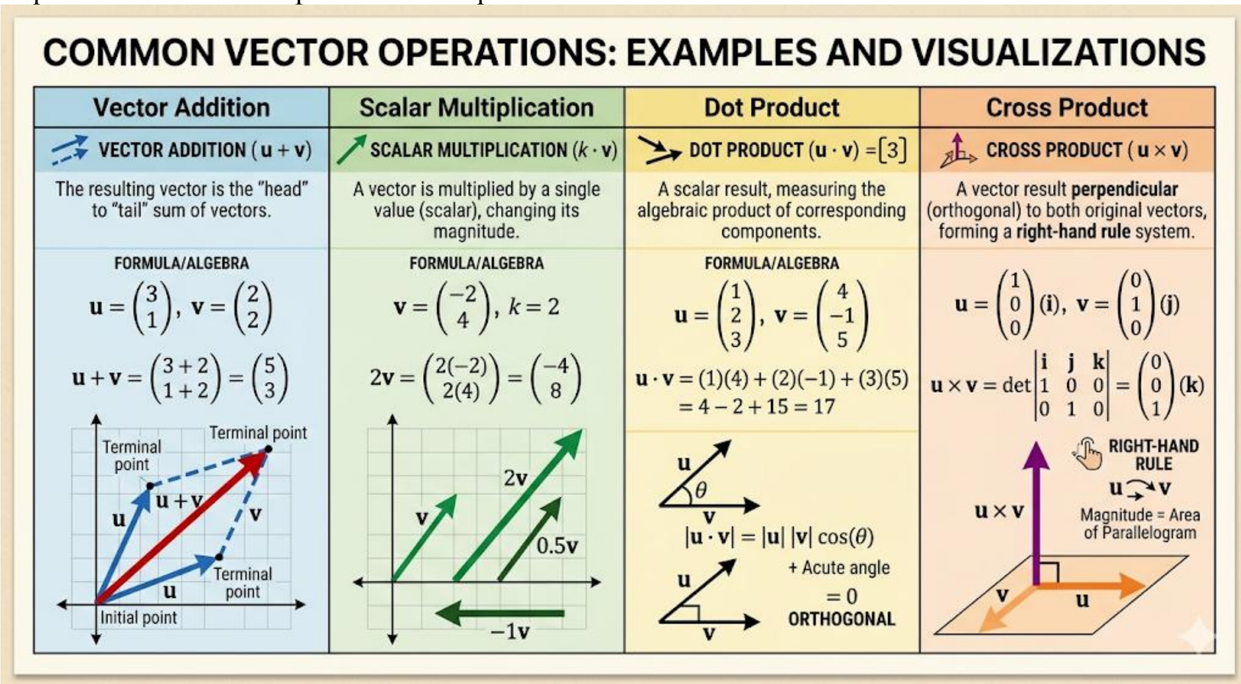
2.3.2 Vector operations (addition, scalar multiplication, dot product, cross product)

Vectors can be manipulated using several operations:

- **Addition:** Adding corresponding components of two vectors.
- **Scalar multiplication:** Multiplying each component by a constant.
- **Dot product:** Produces a scalar, calculated as (\mathbf{a} \cdot \mathbf{b} = |a||b|\cos\theta).
- **Cross product:** Produces a vector perpendicular to both, used in three dimensions.

Fill in the blank: The dot product of two vectors produces a ________.

Caption: Table 2.3 Examples of vector operations



2.3.3 Applications of vectors in mathematics and business

Vectors are used in mathematics to represent physical quantities such as force and velocity. In business, vectors can model data such as sales growth in different regions or resource allocation across departments. They provide a structured way to analyse multidimensional information.

Fill in the blank: In business, vectors can be used to model ________ growth across different regions.

Caption: Box 2.3 Applications of vectors in mathematics and business

Field	**Application**	**Real-World Example**
Mathematics	**Geometric Transformations**	Used in computer graphics to translate, rotate, or scale objects in a 2D or 3D coordinate system.
Mathematics	**Physics & Mechanics**	Representing forces, velocity, and acceleration. For instance, calculating the resultant force when two people pull an object in different directions.

Field	Application	Real-World Example
Business	**Portfolio Management**	Representing a basket of assets (stocks, bonds) as a vector where each component is the quantity or weight of a specific asset.
Business	**Inventory Control**	Storing stock levels of different products across multiple warehouses. A vector $\mathbf{v} = [100, 50, 20]$ could represent 100 units of Item A, 50 of Item B, and 20 of Item C.
Business	**Marketing Analysis**	Customer profiles are often "feature vectors" containing data like age, spending habits, and location to calculate "distance" (similarity) between customers.

Pilot questions 2.3

1. A vector is defined as a quantity with:
   a. Magnitude only
   b. Direction only
   c. Both magnitude and direction
   d. Neither magnitude nor direction
2. The dot product of two vectors results in:
   a. A vector
   b. A scalar
   c. A matrix
   d. A determinant
3. Which vector operation produces a vector perpendicular to both original vectors?
   a. Addition
   b. Scalar multiplication
   c. Dot product
   d. Cross product
4. In business, vectors can be used to represent:
   a. Customer names
   b. Sales growth across regions
   c. Employee attendance
   d. Meeting schedules
5. Which operation involves multiplying each component of a vector by a constant?
   a. Addition
   b. Scalar multiplication
   c. Dot product
   d. Cross product

Series Summary

This Series has introduced you to matrices, determinants, and vectors, which are fundamental tools in mathematics and business applications. We began by defining matrices, exploring their types, and learning how to perform basic operations such as addition, subtraction, and multiplication. We then examined determinants, focusing on their calculation, properties, and applications, particularly in solving systems of equations through methods like Cramer's Rule. Finally, we studied vectors, their representation, operations such as dot and cross products, and their practical applications in both mathematical modelling and business contexts.

By completing this Series, you should now be able to define and manipulate matrices, calculate and apply determinants, and represent and operate with vectors. These skills directly align with the Series Learning Outcomes and provide a strong foundation for tackling more advanced mathematical techniques in subsequent Series.

Glossary of Terms

- **Cross product:** A vector operation that produces a vector perpendicular to two given vectors, used in three dimensions.
- **Determinant:** A scalar value computed from a square matrix, used to determine properties such as invertibility.
- **Diagonal matrix:** A square matrix where all non-diagonal elements are zero.
- **Identity matrix:** A square matrix with ones on the main diagonal and zeros elsewhere.
- **Matrix:** A rectangular arrangement of numbers, symbols, or expressions organised in rows and columns.
- **Scalar multiplication:** Multiplying each element of a matrix or vector by a constant.
- **Square matrix:** A matrix with the same number of rows and columns.
- **Symmetric matrix:** A square matrix that is equal to its transpose.
- **Vector:** A quantity that has both magnitude and direction, often represented as an arrow or ordered pair.
- **Venn diagram (not used here but referenced in Series 1):** A graphical representation of sets using overlapping circles.
- **Zero matrix:** A matrix in which all elements are zero.

Concept Check Questions 2

Objective questions

1. Which type of matrix has ones on the main diagonal and zeros elsewhere? (Linked to SLO 2.1)
2. The determinant of a 2 × 2 matrix (\begin{bmatrix} a & b \ c & d \end{bmatrix}) is calculated as: (Linked to SLO 2.3)
3. The dot product of two vectors results in: (Linked to SLO 2.6)

Short-answer questions

4. Explain the difference between a symmetric matrix and a diagonal matrix. (Linked to SLO 2.1 and SLO 2.3)
5. Define a vector and give one example of its application in business. (Linked to SLO 2.5 and SLO 2.7)

Long-answer questions

6. Demonstrate how determinants can be applied to solve a system of equations using Cramer's Rule, and evaluate its usefulness in business problem-solving. (Linked to SLO 2.4)
7. Analyse how vectors can be used to represent multidimensional business data, such as sales growth across regions, and discuss the advantages of this approach. (Linked to SLO 2.7)

Answers to Concept Check Questions 2

Objective questions

1. Identity matrix
2. (ad - bc)
3. A scalar

Short-answer questions

4. A symmetric matrix is equal to its transpose, meaning its elements are mirrored across the diagonal. A diagonal matrix has all non-diagonal elements equal to zero, with values only on the main diagonal.
5. A vector is a quantity with both magnitude and direction. In business, vectors can represent sales growth across different regions, for example, ((10, 15)) showing growth in two regions.

Long-answer questions

6. *Basic response:* Determinants can be used in Cramer's Rule to solve systems of equations. For example, in a system of two equations with two unknowns, determinants help calculate the values of each variable.
   Value-added response: In business, Cramer's Rule can be applied to resource allocation problems, where equations represent constraints and determinants provide exact solutions. This ensures efficient use of resources and accurate decision-making.
7. *Basic response:* Vectors can represent multidimensional data such as sales growth in different regions, allowing comparisons across multiple dimensions.
   Value-added response: Using vectors enables businesses to visualise and analyse complex data sets, identify trends across regions, and make strategic decisions based on multidimensional insights. This approach enhances clarity and supports data-driven planning.

Answers to Pilot Questions [Series 2]

Part 2.1: Introduction to Matrices

1. b
2. b
3. b
4. a
5. b

Part 2.2: Determinants and Their Applications

1. b
2. b
3. b
4. c
5. b

Part 2.3: Vectors and Their Properties

1. c
2. b
3. d
4. b
5. b

References and Attributions [Series 2]

- Suggested illustration sources:
    - Figure 2.1 Types of matrices: “types of matrices examples OER”
    - Table 2.1 Example of matrix multiplication: “matrix multiplication example OER”
    - Box 2.1 Properties of special matrices: “special matrices properties OER”
    - Table 2.2 Step-by-step calculation of a 3 × 3 determinant: “determinant calculation 3x3 matrix OER”
    - Box 2.2 Properties of determinants: “properties of determinants OER”
    - Figure 2.2 Application of determinants in solving equations: “Cramer’s Rule determinants OER”
    - Figure 2.3 Representation of vectors: “vector representation diagram OER”
    - Table 2.3 Examples of vector operations: “vector operations examples OER”
    - Box 2.3 Applications of vectors in mathematics and business: “applications of vectors in business OER”
- AI prompt suggestions were provided for each illustration to support future generation using AI tools.
- References:
    - Open Educational Resources (OER) in linear algebra and vector mathematics.
    - Standard texts in business mathematics covering matrices, determinants, and vectors.

Series 3: Complex Numbers and Coordinate Geometry

- **Part 3.1: Introduction to Complex Numbers**
    - Sub-Part 3.1.1: Definition and standard form of complex numbers
    - Sub-Part 3.1.2: Basic operations with complex numbers (addition, subtraction, multiplication, division)
    - Sub-Part 3.1.3: Polar form and graphical representation
- **Part 3.2: Properties and Applications of Complex Numbers**
    - Sub-Part 3.2.1: Conjugates and modulus
    - Sub-Part 3.2.2: Powers and roots of complex numbers
    - Sub-Part 3.2.3: Applications of complex numbers in mathematics and business
- **Part 3.3: Coordinate Geometry of Straight Lines**
    - Sub-Part 3.3.1: Equation of a straight line
    - Sub-Part 3.3.2: Slope and intercepts
    - Sub-Part 3.3.3: Applications of straight lines in problem-solving
- **Part 3.4: Coordinate Geometry of Circles**
    - Sub-Part 3.4.1: Equation of a circle
    - Sub-Part 3.4.2: Properties of circles
    - Sub-Part 3.4.3: Applications of circles in mathematics and business

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Introduction

Background
Complex numbers and coordinate geometry are essential tools in mathematics, providing ways to represent and solve problems that go beyond real numbers and simple algebra. Complex numbers extend our number system to include imaginary components, while coordinate geometry allows us to describe shapes and relationships in space using equations. Together, they form a powerful framework for both theoretical and applied problem-solving.

Series Aim
The aim of this Series is to equip you with the ability to work confidently with complex numbers and to apply coordinate geometry in analysing straight lines and circles.

Series Synopsis
We shall commence this Series by introducing complex numbers, their definition, and basic

operations. Next, we will explore their properties, including conjugates, modulus, and applications. Following that, we will move on to coordinate geometry of straight lines, examining equations, slopes, and intercepts. Finally, we will conclude with the coordinate geometry of circles, focusing on their equations, properties, and applications in mathematics and business contexts.

Series Learning Outcomes

Upon completing this Series, you should be able to:

- **SLO 3.1** define complex numbers and express them in standard form,
- **SLO 3.2** perform basic operations with complex numbers,
- **SLO 3.3** represent complex numbers in polar form and graphically,
- **SLO 3.4** explain and apply properties such as conjugates and modulus,
- **SLO 3.5** calculate powers and roots of complex numbers,
- **SLO 3.6** formulate and interpret equations of straight lines,
- **SLO 3.7** analyse slopes and intercepts in coordinate geometry,
- **SLO 3.8** derive and apply equations of circles, and
- **SLO 3.9** evaluate applications of complex numbers and coordinate geometry in mathematics and business contexts.

3.1 Introduction to Complex Numbers

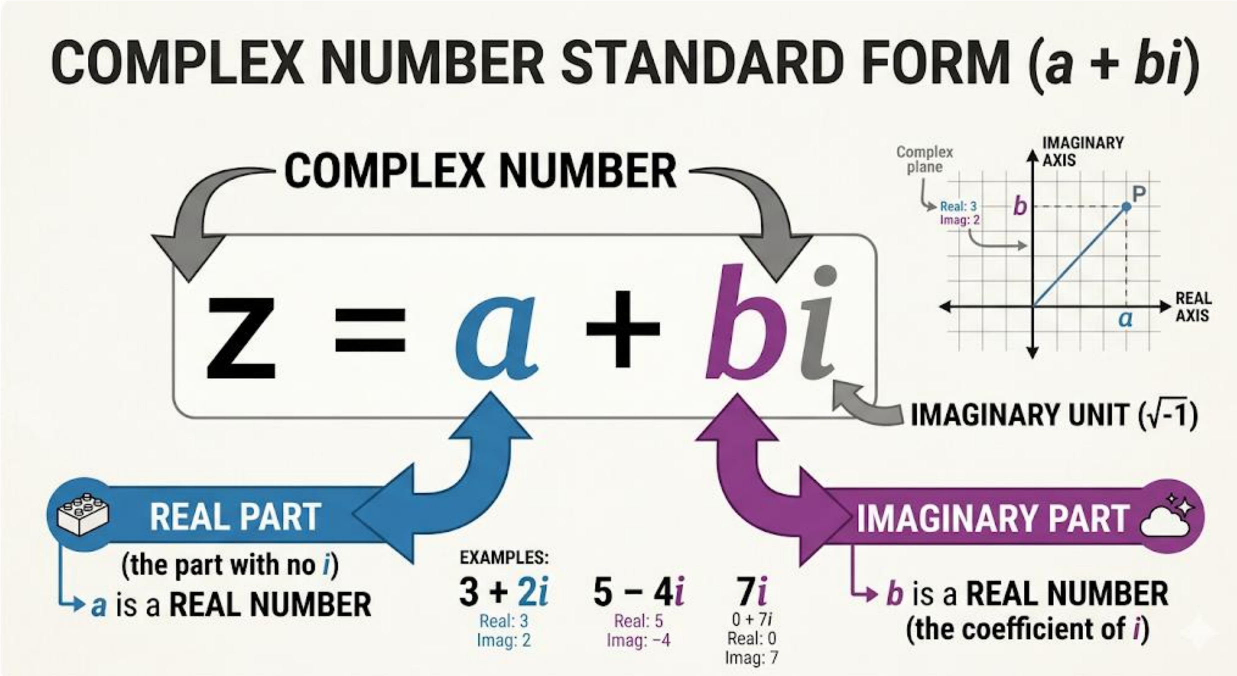
3.1.1 Definition and standard form of complex numbers

A **complex number** is a number that combines a real part and an imaginary part. It is written in the standard form (a + bi), where (a) is the real part and (b) is the imaginary part, with (i) representing the square root of (-1). For example, (3 + 2i) is a complex number with real part 3 and imaginary part 2. Complex numbers extend our number system beyond real numbers, allowing us to solve equations that have no real solutions.

Fill in the blank: A complex number is expressed in the standard form ________.

Diagram showing the standard form of a complex number with labelled real and imaginary parts

Caption: Figure 3.1 Standard form of a complex number



3.1.2 Basic operations with complex numbers

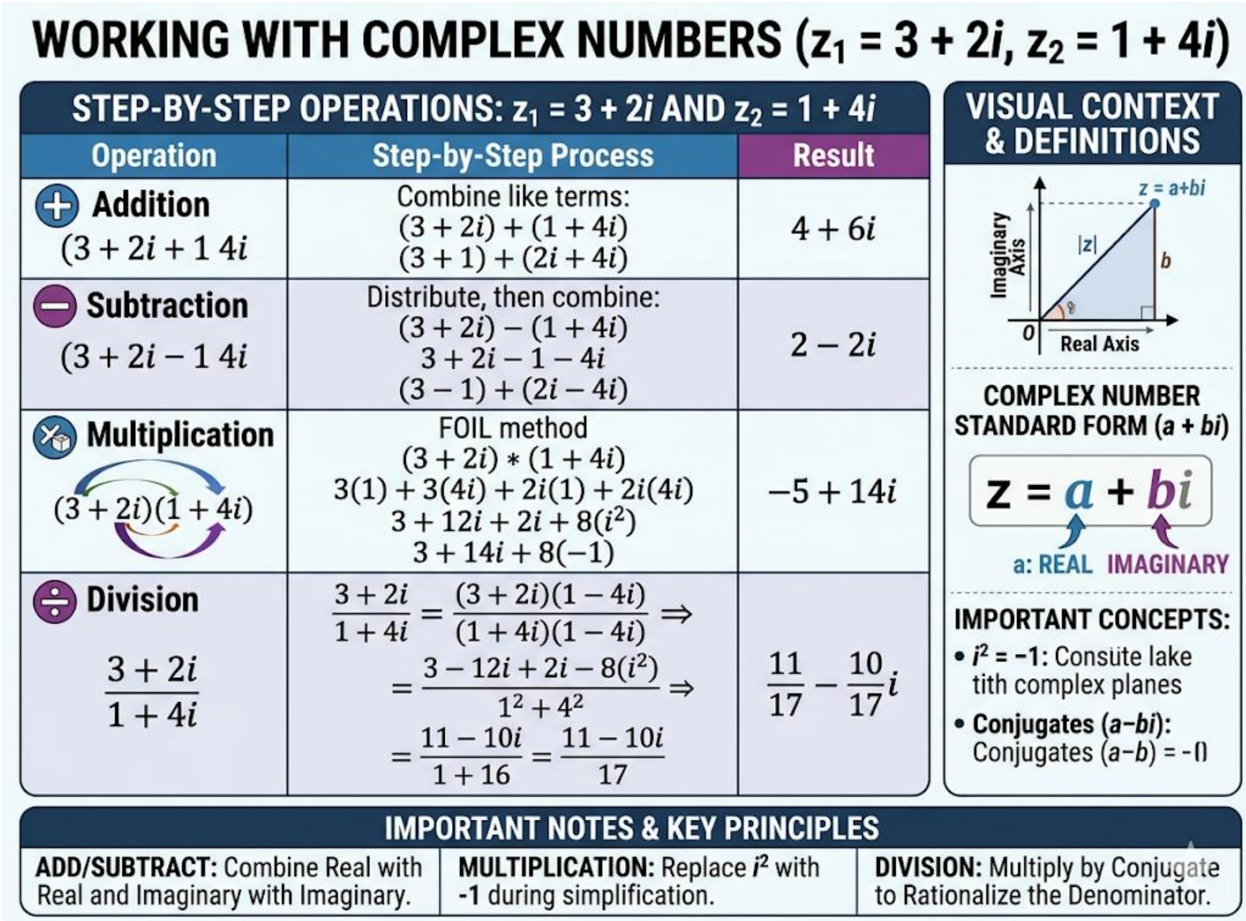
Complex numbers can be manipulated using basic operations:

- **Addition:** Add the real parts together and the imaginary parts together.
- **Subtraction:** Subtract the real parts and the imaginary parts separately.
- **Multiplication:** Apply distributive law and use the rule $(i^2 = -1)$.
- **Division:** Multiply numerator and denominator by the conjugate of the denominator to simplify.

For example, $((3 + 2i) + (1 + 4i) = 4 + 6i)$.

Fill in the blank: When multiplying complex numbers, we use the rule that $(i^2 = \_\_\_\_\_\_\_\_)$.

Caption: Table 3.1 Examples of basic operations with complex numbers



WORKING WITH COMPLEX NUMBERS ($z_1 = 3 + 2i$, $z_2 = 1 + 4i$)

STEP-BY-STEP OPERATIONS: $z_1 = 3 + 2i$ AND $z_2 = 1 + 4i$

Operation	Step-by-Step Process	Result
Addition $(3 + 2i + 1\ 4i$	Combine like terms: $(3 + 2i) + (1 + 4i)$ $(3 + 1) + (2i + 4i)$	$4 + 6i$
Subtraction $(3 + 2i - 1\ 4i$	Distribute, then combine: $(3 + 2i) - (1 + 4i)$ $3 + 2i - 1 - 4i$ $(3 - 1) + (2i - 4i)$	$2 - 2i$
Multiplication $(3 + 2i)(1 + 4i)$	FOIL method $(3 + 2i) * (1 + 4i)$ $3(1) + 3(4i) + 2i(1) + 2i(4i)$ $3 + 12i + 2i + 8(i^2)$ $3 + 14i + 8(-1)$	$-5 + 14i$
Division $\frac{3 + 2i}{1 + 4i}$	$\frac{3+2i}{1+4i} = \frac{(3+2i)(1-4i)}{(1+4i)(1-4i)} \Rightarrow$ $= \frac{3 - 12i + 2i - 8(i^2)}{1^2 + 4^2} \Rightarrow$ $= \frac{11 - 10i}{1 + 16} = \frac{11 - 10i}{17}$	$\frac{11}{17} - \frac{10}{17}i$

IMPORTANT NOTES & KEY PRINCIPLES

ADD/SUBTRACT	MULTIPLICATION	DIVISION
ADD/SUBTRACT: Combine Real with Real and Imaginary with Imaginary.	**MULTIPLICATION:** Replace i^2 with -1 during simplification.	**DIVISION:** Multiply by Conjugate to Rationalize the Denominator.

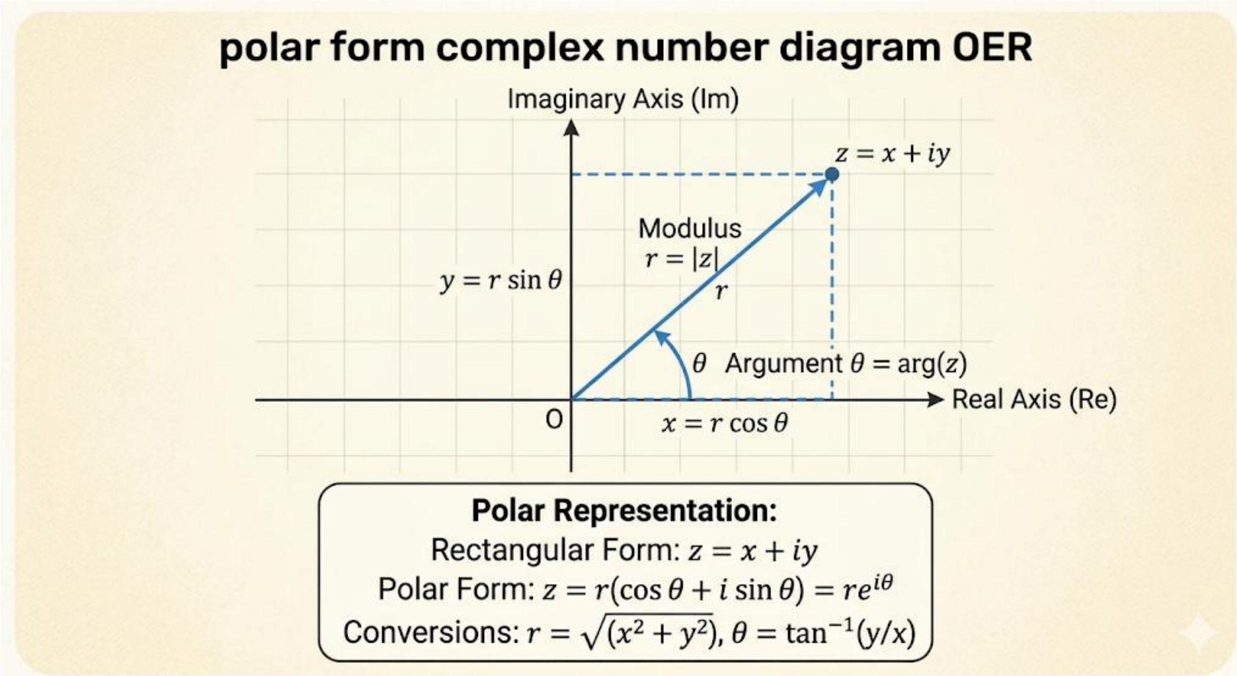
3.1.3 Polar form and graphical representation

Complex numbers can also be expressed in **polar form**, written as (r(\cos\theta + i\sin\theta)), where (r) is the modulus (distance from the origin) and (\theta) is the argument (angle with the positive x-axis). This form is useful for multiplication and division, as well as for representing complex numbers graphically on the complex plane.

Fill in the blank: In polar form, a complex number is expressed as (r(\cos\theta + i\sin\theta)), where (r) is the ________ and (\theta) is the argument.

Diagram showing a complex number represented on the complex plane with modulus and argument

Caption: Figure 3.2 Polar representation of a complex number



Pilot questions 3.1

1. Which of the following is a complex number in standard form?
   a. (5)
   b. (3 + 2i)
   c. (i^2)
   d. (\sqrt{2})

2. In the complex number (7 - 4i), the real part is:
   a. 7
   b. -4
   c. i
   d. None

3. The rule used in multiplication of complex numbers is:
   a. (i^2 = 1)
   b. (i^2 = -1)
   c. (i^2 = 0)
   d. (i^2 = 2)

4. In polar form, the modulus of a complex number represents:
   a. The angle with the x-axis
   b. The distance from the origin
   c. The imaginary part
   d. The real part

5. Which form of complex numbers is most useful for multiplication and division?
   a. Standard form
   b. Polar form
   c. Real form
   d. Imaginary form

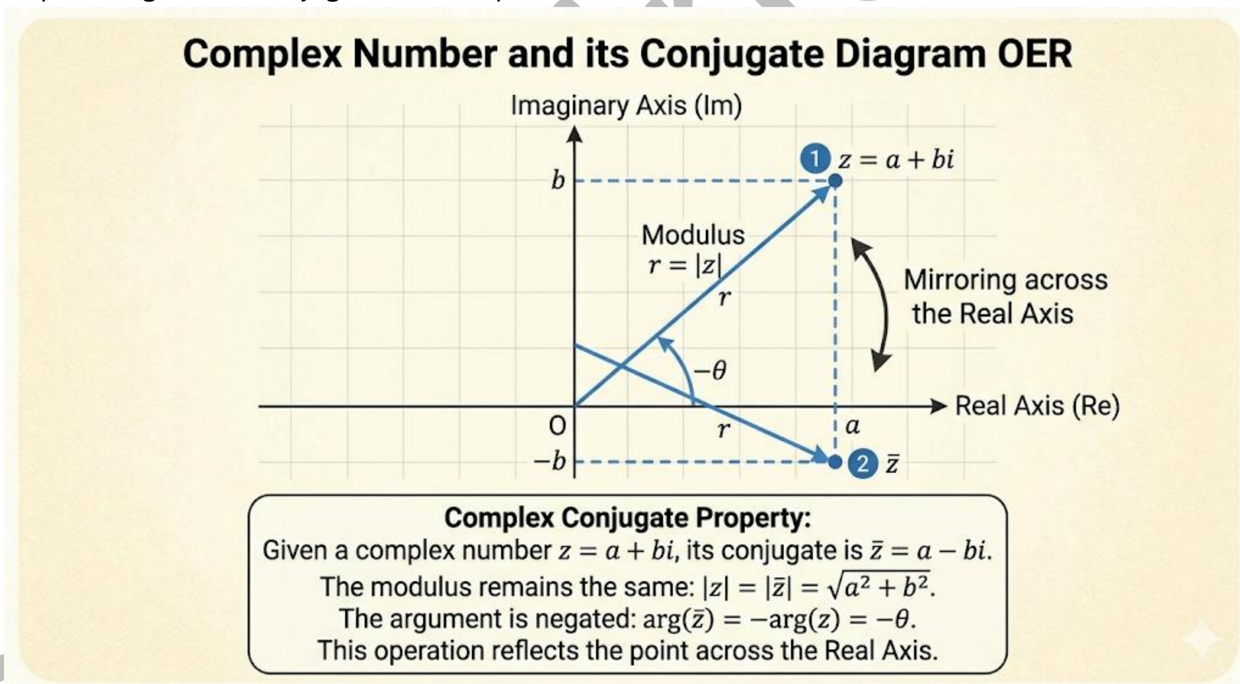
3.2 Properties and Applications of Complex Numbers

3.2.1 Conjugates and modulus

The **conjugate** of a complex number (a + bi) is (a - bi). Conjugates are useful in simplifying division of complex numbers. The **modulus** of a complex number is its distance from the origin on the complex plane, calculated as (\sqrt{a^2 + b^2}). For example, the modulus of (3 + 4i) is 5.

Fill in the blank: The conjugate of (a + bi) is ________.

Diagram showing a complex number and its conjugate on the complex plane
Caption: Figure 3.3 Conjugate of a complex number

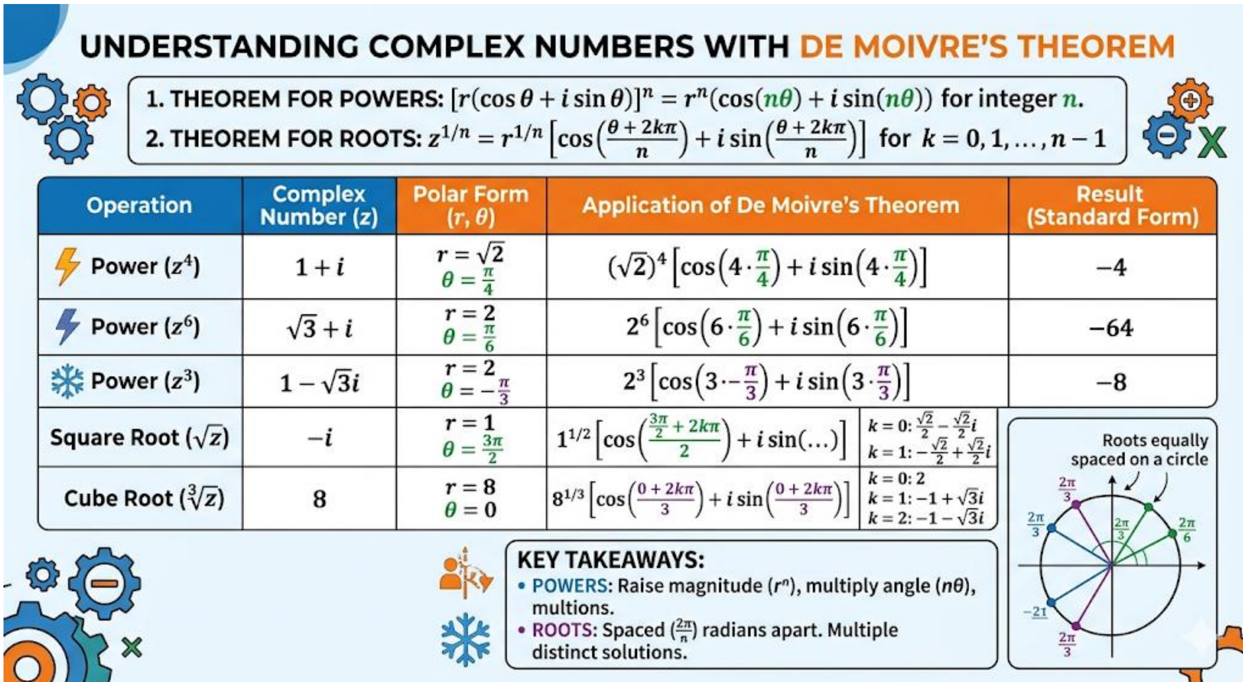


3.2.2 Powers and roots of complex numbers

Complex numbers can be raised to powers or expressed in roots using **De Moivre's Theorem**, which states:
[(r(\cos\theta + i\sin\theta))^n = r^n(\cos n\theta + i\sin n\theta)]
This theorem simplifies calculations involving powers and roots of complex numbers in polar form.

Fill in the blank: De Moivre's Theorem is used to calculate ________ and roots of complex numbers.

Caption: Table 3.2 Examples of powers and roots using De Moivre's Theorem



Operation	Complex Number (z)	Polar Form (r, θ)	Application of De Moivre's Theorem	Result (Standard Form)
Power (z^4)	$1+i$	$r=\sqrt{2}$ $\theta=\frac{\pi}{4}$	$(\sqrt{2})^4\left[\cos\left(4\cdot\frac{\pi}{4}\right)+i\sin\left(4\cdot\frac{\pi}{4}\right)\right]$	-4
Power (z^6)	$\sqrt{3}+i$	$r=2$ $\theta=\frac{\pi}{6}$	$2^6\left[\cos\left(6\cdot\frac{\pi}{6}\right)+i\sin\left(6\cdot\frac{\pi}{6}\right)\right]$	-64
Power (z^3)	$1-\sqrt{3}i$	$r=2$ $\theta=-\frac{\pi}{3}$	$2^3\left[\cos\left(3\cdot-\frac{\pi}{3}\right)+i\sin\left(3\cdot\frac{\pi}{3}\right)\right]$	-8
Square Root ($\sqrt{z}$)	$-i$	$r=1$ $\theta=\frac{3\pi}{2}$	$1^{1/2}\left[\cos\left(\frac{\frac{3\pi}{2}+2k\pi}{2}\right)+i\sin(\ldots)\right]$	$k=0: \frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$ $k=1: -\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$
Cube Root ($\sqrt[3]{z}$)	8	$r=8$ $\theta=0$	$8^{1/3}\left[\cos\left(\frac{0+2k\pi}{3}\right)+i\sin\left(\frac{0+2k\pi}{3}\right)\right]$	$k=0: 2$ $k=1: -1+\sqrt{3}i$ $k=2: -1-\sqrt{3}i$

3.2.3 Applications of complex numbers in mathematics and business

Complex numbers are applied in mathematics to solve equations without real solutions, such as quadratic equations with negative discriminants. In business and engineering, they are used in modelling electrical circuits, analysing signals, and solving optimisation problems.

Fill in the blank: Complex numbers are used in business and engineering to model ________ circuits.

Caption: Box 3.1 Applications of complex numbers in mathematics and business

Field	Key Applications
Mathematics	Used to solve **polynomial equations** that have no real solutions (e.g., $x^2 + 1 = 0$). They are fundamental to **Fractal Geometry** (like the Mandelbrot set) and **Complex Analysis**, which simplifies the integration of difficult real-valued functions.
Engineering	Essential in **Electrical Engineering** for analyzing **AC (Alternating Current) circuits**, where "impedance" is represented as a complex number. They are also used in **Signal Processing** (Fourier Transforms) for

Field	Key Applications
	telecommunications, audio compression, and **Control Theory** to ensure the stability of mechanical systems.
Physics	In **Quantum Mechanics**, the state of a particle (the wave function ψ) is inherently complex. In **Fluid Dynamics**, complex potential functions are used to model the flow of air over wings or water through pipes.
Business & Economics	Used in complex **Stochastic Modeling** to predict market fluctuations and in **Signal Analysis** for high-frequency trading algorithms. They also appear in specific types of **optimization problems** and structural modeling of cyclical economic trends.

Pilot questions 3.2

1. The conjugate of (5 + 3i) is:
   a. (5 - 3i)
   b. (-5 + 3i)
   c. (5 + 3i)
   d. (-5 - 3i)
2. The modulus of (6 + 8i) is:
   a. 10
   b. 14
   c. 12
   d. 8
3. De Moivre's Theorem is used to calculate:
   a. Addition of complex numbers
   b. Powers and roots of complex numbers
   c. Subtraction of complex numbers
   d. Division of complex numbers
4. Complex numbers are applied in solving:
   a. Equations with negative discriminants
   b. Equations with positive discriminants
   c. Equations with zero discriminants
   d. Equations with no variables

5. In business, complex numbers are used to model:
   a. Customer behaviour
   b. Electrical circuits
   c. Employee attendance
   d. Market surveys

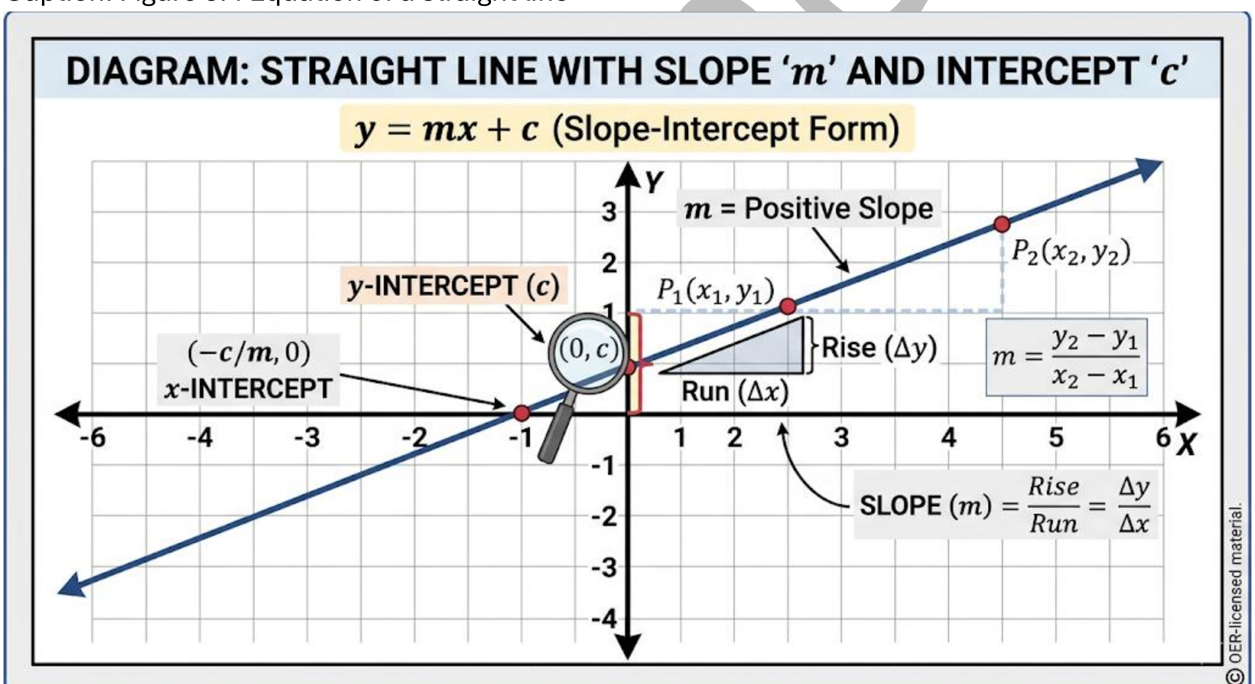
3.3 Coordinate Geometry of Straight Lines

3.3.1 Equation of a straight line

The general equation of a straight line is (y = mx + c), where (m) is the slope and (c) is the y-intercept. This equation allows us to represent linear relationships graphically.

Fill in the blank: In the equation (y = mx + c), (m) represents the ________.

Diagram showing a straight line with slope and intercepts
Caption: Figure 3.4 Equation of a straight line

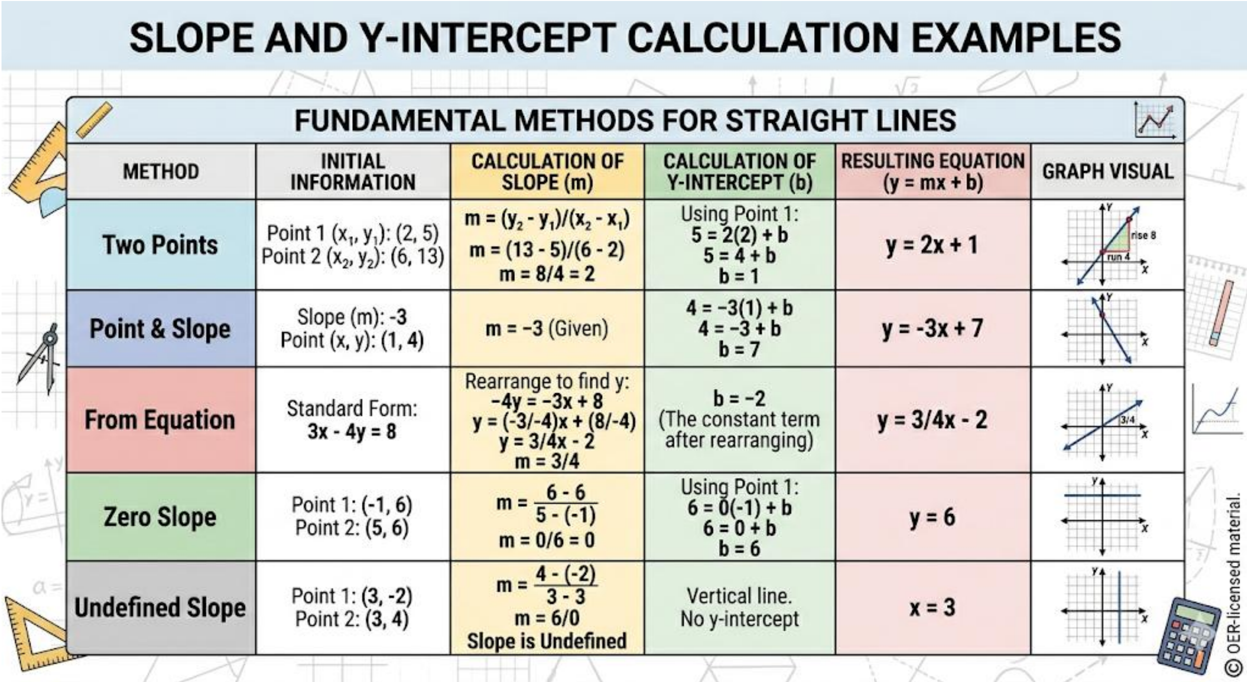


3.3.2 Slope and intercepts

The **slope** of a line measures its steepness, calculated as the change in y divided by the change in x. The **intercepts** are the points where the line crosses the axes. These features are essential in analysing linear relationships.

Fill in the blank: The slope of a line is calculated as change in y divided by change in ________.

Caption: Table 3.3 Examples of slope and intercept calculations



SLOPE AND Y-INTERCEPT CALCULATION EXAMPLES

FUNDAMENTAL METHODS FOR STRAIGHT LINES

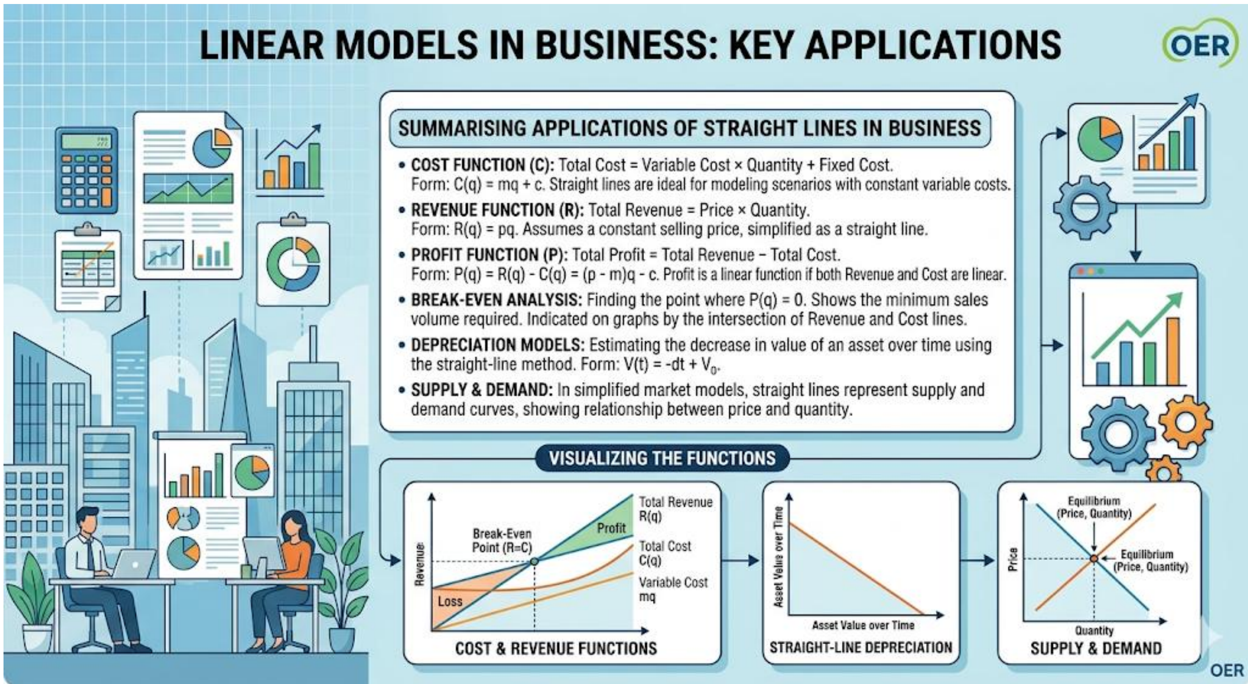
METHOD	INITIAL INFORMATION	CALCULATION OF SLOPE (m)	CALCULATION OF Y-INTERCEPT (b)	RESULTING EQUATION (y = mx + b)	GRAPH VISUAL
Two Points	Point 1 (x_1, y_1): (2, 5) Point 2 (x_2, y_2): (6, 13)	$m = (y_2 - y_1)/(x_2 - x_1)$ m = (13 - 5)/(6 - 2) m = 8/4 = 2	Using Point 1: 5 = 2(2) + b 5 = 4 + b b = 1	y = 2x + 1	
Point & Slope	Slope (m): -3 Point (x, y): (1, 4)	m = −3 (Given)	4 = −3(1) + b 4 = −3 + b b = 7	y = -3x + 7	
From Equation	Standard Form: 3x - 4y = 8	Rearrange to find y: −4y = −3x + 8 y = (-3/-4)x + (8/-4) y = 3/4x - 2 m = 3/4	b = -2 (The constant term after rearranging)	y = 3/4x - 2	
Zero Slope	Point 1: (-1, 6) Point 2: (5, 6)	$m = \frac{6 - 6}{5 - (-1)}$ m = 0/6 = 0	Using Point 1: 6 = 0(-1) + b 6 = 0 + b b = 6	y = 6	
Undefined Slope	Point 1: (3, -2) Point 2: (3, 4)	$m = \frac{4 - (-2)}{3 - 3}$ m = 6/0 Slope is Undefined	Vertical line. No y-intercept	x = 3	

3.3.3 Applications of straight lines in problem-solving

Straight lines are used in business mathematics to model cost functions, revenue functions, and break-even analysis. They provide a simple yet powerful way to represent linear relationships.

Fill in the blank: Straight lines are used in business mathematics to model ________ analysis.

Caption: Box 3.2 Applications of straight lines in business mathematics



Pilot questions 3.3

1. In the equation (y = mx + c), (c) represents:
   a. Slope
   b. x-intercept
   c. y-intercept
   d. Gradient
2. The slope of a line is calculated as:
   a. Change in x divided by change in y
   b. Change in y divided by change in x
   c. Product of x and y
   d. Difference between x and y
3. Straight lines are used in business mathematics to model:
   a. Break-even analysis
   b. Customer satisfaction
   c. Employee attendance
   d. Market surveys
4. The slope of a horizontal line is:
   a. 0
   b. 1

c. Undefined
d. Infinite

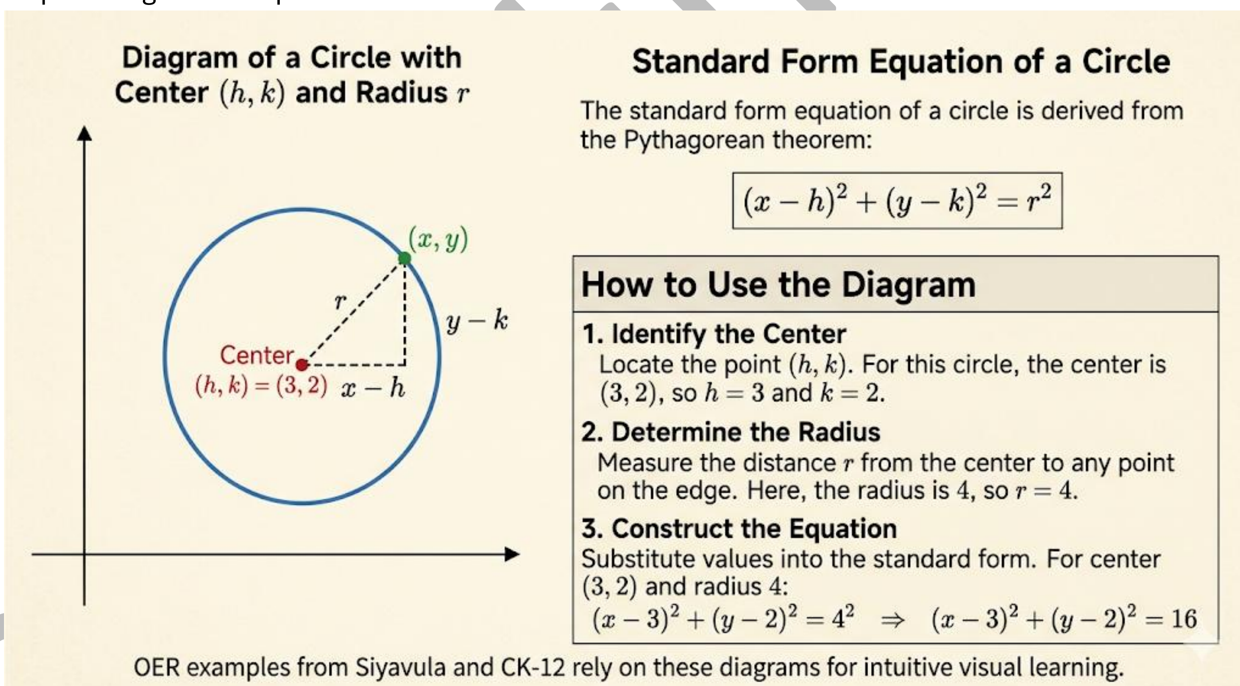
5. The slope of a vertical line is:
 a. 0
 b. 1
 c. Undefined
 d. Infinite

3.4 Coordinate Geometry of Circles

3.4.1 Equation of a circle

The general equation of a circle with centre ((h, k)) and radius (r) is:
[(x - h)^2 + (y - k)^2 = r^2]
This equation allows us to represent circles on the coordinate plane.

Diagram showing a circle with centre and radius labelled
Caption: Figure 3.5 Equation of a circle



3.4.2 Properties of circles

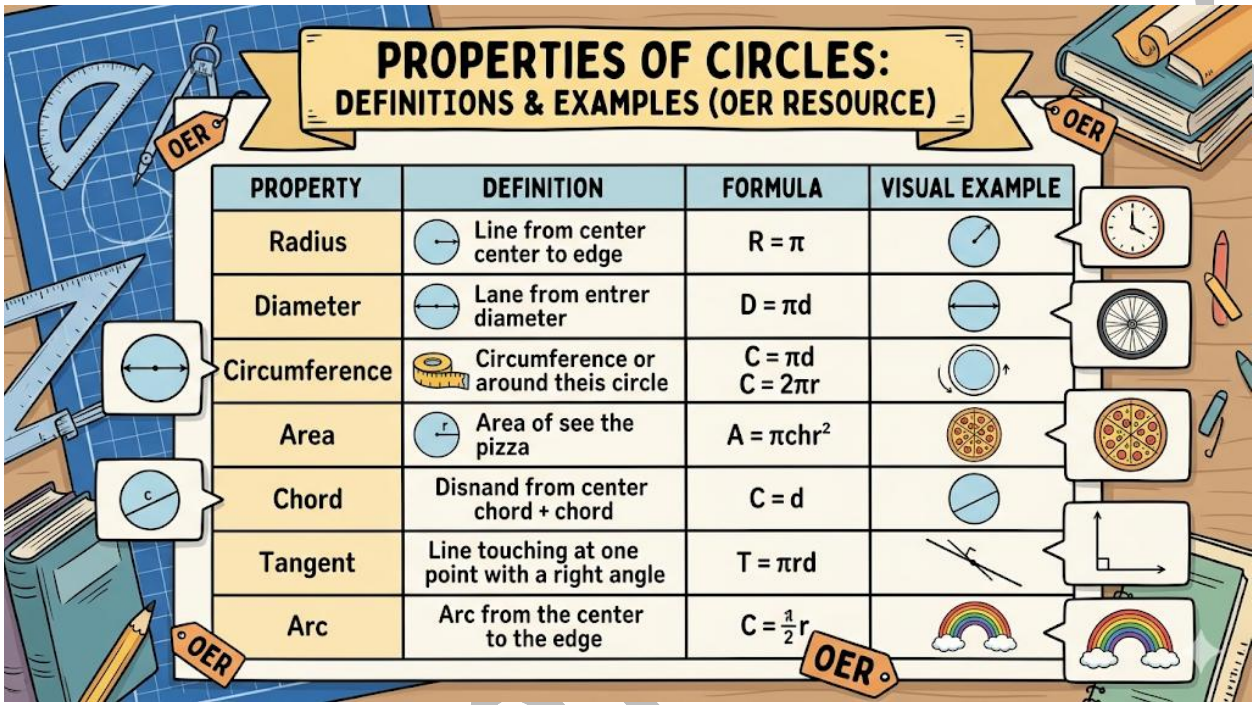
Circles have several important properties:

- All points on the circle are equidistant from the centre.
- The diameter is twice the radius.

- The tangent to a circle is perpendicular to the radius at the point of contact.

Fill in the blank: The tangent to a circle is perpendicular to the ________ at the point of contact.

Caption: Table 3.4 Properties of circles



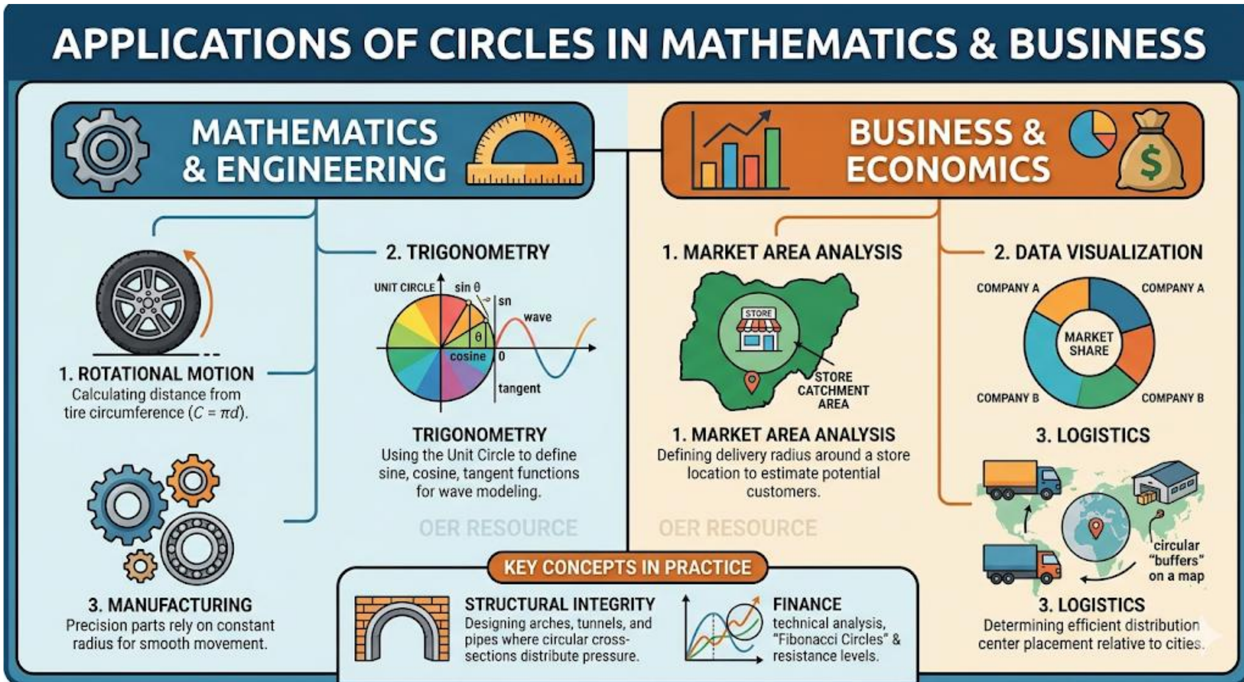
PROPERTY	DEFINITION	FORMULA	VISUAL EXAMPLE
Radius	Line from center center to edge	$R = \pi$	
Diameter	Lane from entrer diameter	$D = \pi d$	
Circumference	Circumference or around theis circle	$C = \pi d$ $C = 2\pi r$	
Area	Area of see the pizza	$A = \pi chr^2$	
Chord	Disnand from center chord + chord	$C = d$	
Tangent	Line touching at one point with a right angle	$T = \pi rd$	
Arc	Arc from the center to the edge	$C = \frac{1}{2}r$	

3.4.3 Applications of circles in mathematics and business

Circles are used in mathematics to study geometry and trigonometry. In business, circular models can represent cycles such as economic trends, production processes, and market behaviour.

Fill in the blank: In business, circles can be used to represent economic ________.

Caption: Box 3.3 Applications of circles in mathematics and business



Pilot questions 3.4

1. The general equation of a circle with centre ((0, 0)) and radius 5 is:
   a. (x^2 + y^2 = 25)
   b. (x^2 + y^2 = 5)
   c. ((x - 5)^2 + (y - 5)^2 = 25)
   d. (x^2 + y^2 = 10)

2. The diameter of a circle is:
   a. Equal to the radius
   b. Twice the radius
   c. Half the radius
   d. Three times the radius

3. The tangent to a circle is:
   a. Parallel to the radius
   b. Perpendicular to the radius
   c. Equal to

Series Summary

This Series has introduced you to complex numbers and coordinate geometry, two essential areas of mathematics that extend problem-solving beyond real numbers and simple algebra. We began with complex numbers, learning their definition, standard form, and basic operations, before exploring their polar representation. We then examined properties such as conjugates and modulus, and applied De Moivre's Theorem to powers and roots, highlighting practical applications in mathematics, engineering, and business. Moving on, we studied straight lines, focusing on their equations, slopes, and intercepts, and concluded with circles, covering their equations, properties, and applications in modelling cycles and trends.

By completing this Series, you should now be able to define, operate, and represent complex numbers, apply their properties in problem-solving, formulate and interpret equations of straight lines, and derive and apply equations of circles. These skills directly align with the Series Learning Outcomes and provide a strong foundation for advanced mathematical and business applications.

Glossary of Terms

- **Argument (θ):** The angle a complex number makes with the positive x-axis in polar form.
- **Complex number:** A number expressed in the form ($a + bi$), combining a real part and an imaginary part.
- **Conjugate:** The complex number obtained by changing the sign of the imaginary part, e.g., ($a + bi$) becomes ($a - bi$).
- **De Moivre's Theorem:** A formula used to calculate powers and roots of complex numbers in polar form.
- **Imaginary unit (i):** Defined as the square root of (-1).
- **Modulus (r):** The distance of a complex number from the origin, calculated as ($\sqrt{a^2 + b^2}$).
- **Polar form:** Representation of a complex number as ($r(\cos\theta + i\sin\theta)$).
- **Slope (m):** A measure of the steepness of a straight line, calculated as change in y divided by change in x.
- **Straight line equation:** The general form ($y = mx + c$), where (m) is slope and (c) is y-intercept.
- **Circle equation:** The general form ($(x - h)^2 + (y - k)^2 = r^2$), representing a circle with centre ((h, k)) and radius (r).
- **Tangent:** A line that touches a circle at exactly one point and is perpendicular to the radius at that point.

Concept Check Questions 3

Objective questions

1. Which of the following is a complex number in standard form? (Linked to SLO 3.1)
2. The rule used in multiplication of complex numbers is: (Linked to SLO 3.2)
3. In polar form, the modulus of a complex number represents: (Linked to SLO 3.3)
4. The slope of a line is calculated as: (Linked to SLO 3.7)
5. The tangent to a circle is always: (Linked to SLO 3.8)

Short-answer questions

6. Define the conjugate of a complex number and explain its use in division. (Linked to SLO 3.4)
7. State De Moivre's Theorem and give one example of its application. (Linked to SLO 3.5)
8. Write the general equation of a straight line and explain the meaning of its parameters. (Linked to SLO 3.6)

Long-answer questions

9. Demonstrate how complex numbers can be represented in polar form and evaluate the advantages of this representation for multiplication and division. (Linked to SLO 3.3)
10. Analyse how straight lines and circles can be applied in business mathematics, using examples such as break-even analysis and economic cycles. (Linked to SLO 3.9)

Answers to Concept Check Questions 3

Objective questions

1. ($3 + 2i$)
2. ($i^2 = -1$)
3. The distance from the origin
4. Change in y divided by change in x
5. Perpendicular to the radius

Short-answer questions

6. The conjugate of ($a + bi$) is ($a - bi$). It is used in division to rationalise denominators and simplify expressions.
7. De Moivre's Theorem states that ($(r(\cos\theta + i\sin\theta))^n = r^n(\cos n\theta + i\sin n\theta)$). For example, ($(1(\cos 90° + i\sin 90°))^2 = -1$).

8. The general equation of a straight line is (y = mx + c), where (m) is the slope (steepness) and (c) is the y-intercept (point where the line crosses the y-axis).

Long-answer questions

9. *Basic response:* Complex numbers can be represented in polar form as (r(\cos\theta + i\sin\theta)). This makes multiplication and division easier, as moduli are multiplied or divided and arguments are added or subtracted.
   Value-added response: Polar form is particularly useful in engineering and physics, where complex numbers represent oscillations and waves. It simplifies calculations involving rotations and periodic functions.

10. *Basic response:* Straight lines can model cost and revenue functions, while circles can represent cycles such as production or economic trends.
    Value-added response: In business, straight lines are used in break-even analysis to determine profitability, while circles can model repeating behaviours such as seasonal demand. These applications help managers make informed decisions and plan strategically.

Answers to Pilot Questions [Series 3]

Part 3.1: Introduction to Complex Numbers

1. b
2. a
3. b
4. b
5. b

Part 3.2: Properties and Applications of Complex Numbers

1. a
2. a
3. b
4. a
5. b

Part 3.3: Coordinate Geometry of Straight Lines

1. c

2. b
3. a
4. a
5. c

Part 3.4: Coordinate Geometry of Circles

1. a
2. b
3. b

References and Attributions [Series 3]

- Suggested illustration sources:
  - Figure 3.1 Standard form of a complex number: “complex number standard form diagram OER”
  - Table 3.1 Examples of basic operations: “complex number operations examples OER”
  - Figure 3.2 Polar representation: “polar form complex number diagram OER”
  - Figure 3.3 Conjugate of a complex number: “complex number conjugate diagram OER”
  - Table 3.2 Powers and roots using De Moivre’s Theorem: “De Moivre’s theorem examples OER”
  - Box 3.1 Applications of complex numbers: “applications of complex numbers OER”
  - Figure 3.4 Equation of a straight line: “equation of straight line diagram OER”
  - Table 3.3 Slope and intercept calculations: “slope and intercept examples OER”
  - Box 3.2 Applications of straight lines in business: “applications of straight lines in business OER”
  - Figure 3.5 Equation of a circle: “equation of circle diagram OER”
  - Table 3.4 Properties of circles: “properties of circles OER”
  - Box 3.3 Applications of circles in business: “applications of circles in business OER”

- AI prompt suggestions were provided for each illustration to support future generation using AI tools.
- References:
    - Open Educational Resources (OER) in complex numbers and coordinate geometry.
    - Standard texts in business mathematics covering algebra, geometry, and applications.

Series 4: Sequences, Series, and Calculus,

- **Part 4.1: Sequences and Their Properties**
    - Sub-Part 4.1.1: Definition and types of sequences
    - Sub-Part 4.1.2: Arithmetic sequences
    - Sub-Part 4.1.3: Geometric sequences
- **Part 4.2: Series and Summation Techniques**
    - Sub-Part 4.2.1: Definition of a series
    - Sub-Part 4.2.2: Arithmetic series and formulae
    - Sub-Part 4.2.3: Geometric series and convergence
- **Part 4.3: Limits and Continuity**
    - Sub-Part 4.3.1: Concept of limits
    - Sub-Part 4.3.2: Evaluating limits
    - Sub-Part 4.3.3: Continuity and its importance
- **Part 4.4: Differentiation and Integration**
    - Sub-Part 4.4.1: Basic rules of differentiation
    - Sub-Part 4.4.2: Applications of differentiation (maximum and minimum points)
    - Sub-Part 4.4.3: Basic rules of integration
    - Sub-Part 4.4.4: Applications of integration in mathematics and business

This gives us **four main Parts**, which conforms to the guideline for a 2-credit course.

Introduction

Background

Sequences, series, and calculus are central to advanced mathematics and play a vital role in business applications. Sequences and series help us describe patterns and cumulative totals, while calculus provides tools for analysing change, growth, and optimisation. These concepts are widely used in economics, finance, and management decision-making.

Series Aim

The aim of this Series is to introduce you to sequences, series, and calculus, and to equip you with the skills needed to analyse patterns, evaluate limits, and apply differentiation and integration in mathematical and business contexts.

Series Synopsis

We shall commence this Series by exploring sequences, focusing on their definitions and types, including arithmetic and geometric sequences. Next, we will move on to series, examining summation techniques and convergence. Following that, we will study limits and continuity, which form the foundation of calculus. Finally, we will conclude with differentiation and integration, highlighting their rules and applications in mathematics and business problem-solving.

Series Learning Outcomes

Upon completing this Series, you should be able to:

- **SLO 4.1** define sequences and distinguish between arithmetic and geometric types,
- **SLO 4.2** calculate terms of arithmetic and geometric sequences,
- **SLO 4.3** explain the concept of a series and apply summation techniques,
- **SLO 4.4** evaluate arithmetic and geometric series using formulae,
- **SLO 4.5** define limits and demonstrate how to evaluate them,
- **SLO 4.6** analyse continuity and explain its importance in calculus,
- **SLO 4.7** apply rules of differentiation to solve problems involving maximum and minimum points,
- **SLO 4.8** perform basic integration and explain its applications, and
- **SLO 4.9** evaluate practical applications of sequences, series, and calculus in mathematics and business contexts.

4.1 Sequences and Their Properties

4.1.1 Definition and types of sequences

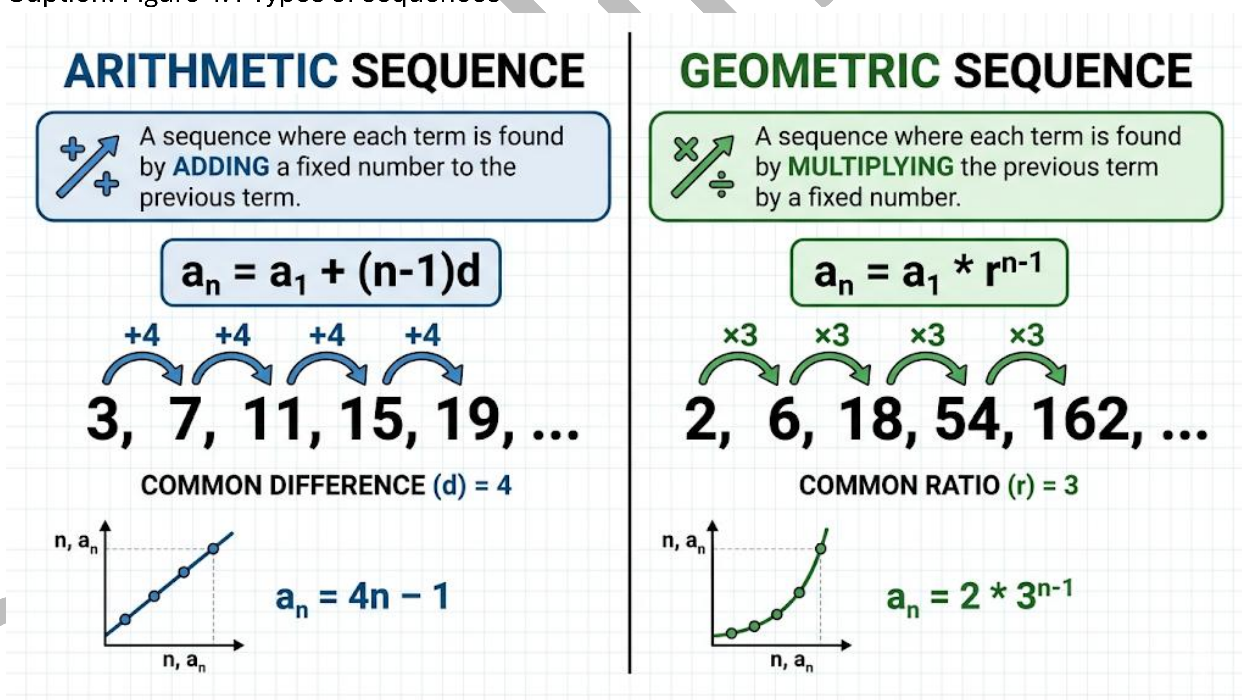
A **sequence** is an ordered list of numbers that follow a specific rule or pattern. Each number in the sequence is called a **term**, and the position of the term is important. Sequences can be finite, with a limited number of terms, or infinite, continuing indefinitely.

Types of sequences include:

- **Arithmetic sequence:** Each term is obtained by adding a fixed number (called the common difference) to the previous term.
- **Geometric sequence:** Each term is obtained by multiplying the previous term by a fixed number (called the common ratio).
- **Other sequences:** Such as Fibonacci sequence, where each term is the sum of the two preceding terms.

Fill in the blank: A sequence in which each term is obtained by adding a fixed number to the previous term is called an ________ sequence.

Diagram showing examples of arithmetic and geometric sequences
Caption: Figure 4.1 Types of sequences



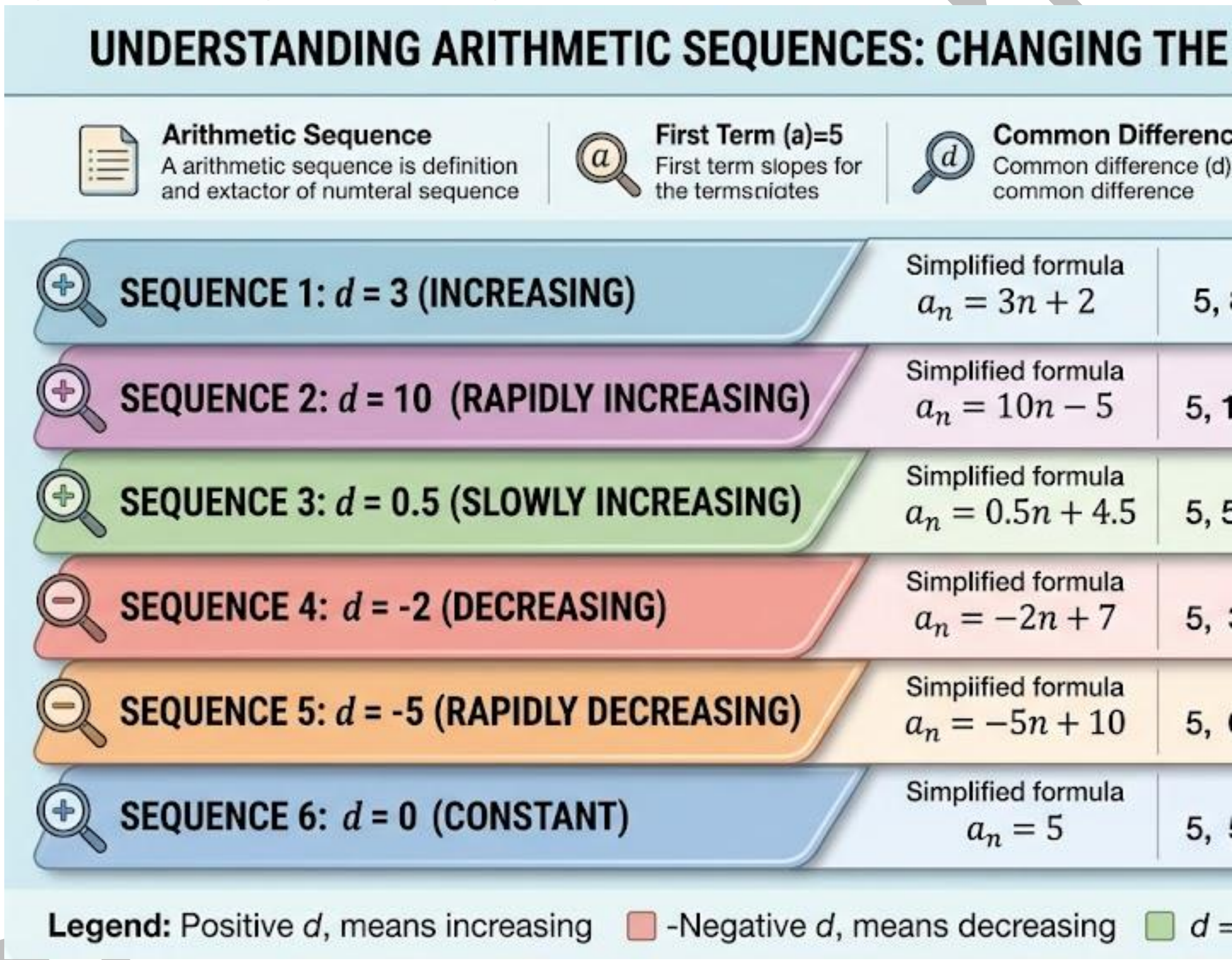
4.1.2 Arithmetic sequences

An **arithmetic sequence** has the general form:
[a, a + d, a + 2d, a + 3d, \ldots]
where (a) is the first term and (d) is the common difference. The nth term of an arithmetic sequence

is given by:
[T_n = a + (n - 1)d]

Fill in the blank: The nth term of an arithmetic sequence is given by (T_n = a + (n - 1) \times ________).

Caption: Table 4.1 Examples of arithmetic sequences with different common differences



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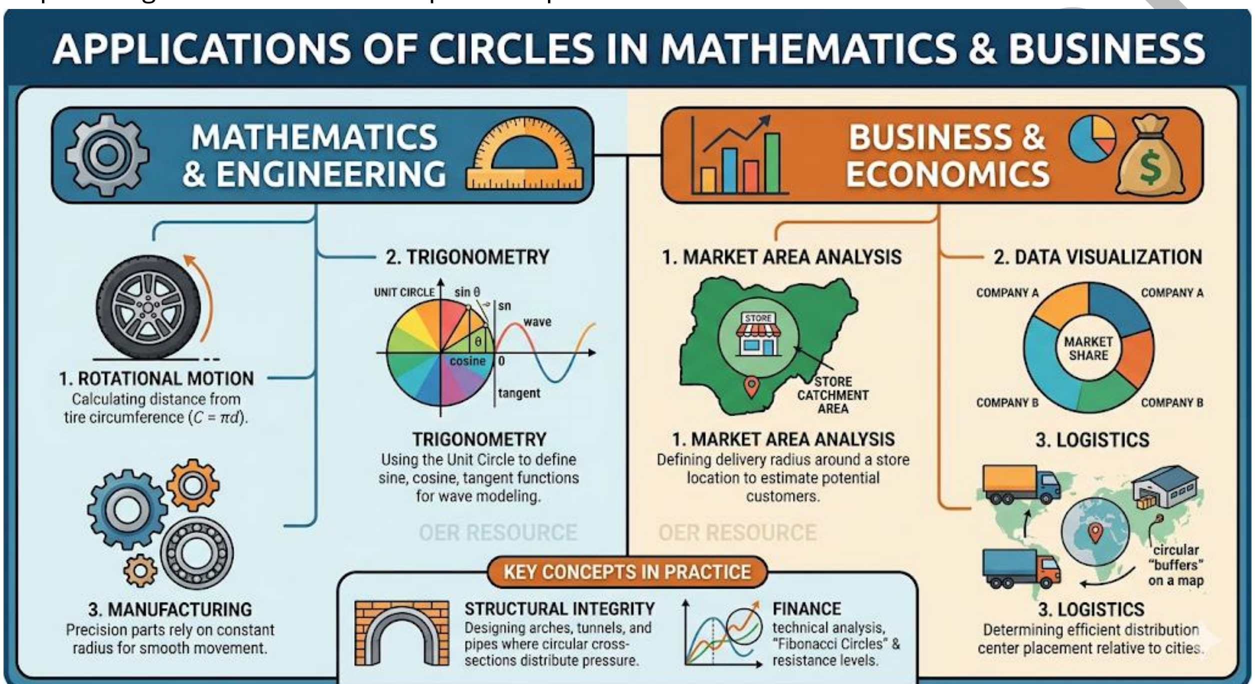
4.1.3 Geometric sequences

A **geometric sequence** has the general form:
[a, ar, ar^2, ar^3, \ldots]
where (a) is the first term and (r) is the common ratio. The nth term of a geometric sequence is given

by:
$$T_n = ar^{n-1}$$

Fill in the blank: The nth term of a geometric sequence is given by ($T_n = a \times r^{n-1}$), where (r) is the ________.

Diagram showing geometric sequence growth with different ratios
Caption: Figure 4.2 Geometric sequence representation



Pilot questions 4.1

1. Which of the following is an arithmetic sequence?
   a. 2, 4, 6, 8, ...
   b. 3, 6, 12, 24, ...
   c. 1, 1, 2, 3, 5, ...
   d. 10, 100, 1000, ...
2. In the arithmetic sequence 5, 10, 15, 20, ... the common difference is:
   a. 2
   b. 5
   c. 10
   d. 15
3. The nth term of an arithmetic sequence is given by:
   a. ($T_n = a + (n - 1)d$)
   b. ($T_n = ar^{n-1}$)

c. ($T_n = a - (n - 1)d$)
d. ($T_n = a + nd$)

4. In the geometric sequence 2, 6, 18, 54, ... the common ratio is:
   a. 2
   b. 3
   c. 6
   d. 18

5. The nth term of a geometric sequence is given by:
   a. ($T_n = a + (n - 1)d$)
   b. ($T_n = ar^{n-1}$)
   c. ($T_n = a - (n - 1)d$)
   d. ($T_n = a + nd$)

4.2 Series and Summation Techniques

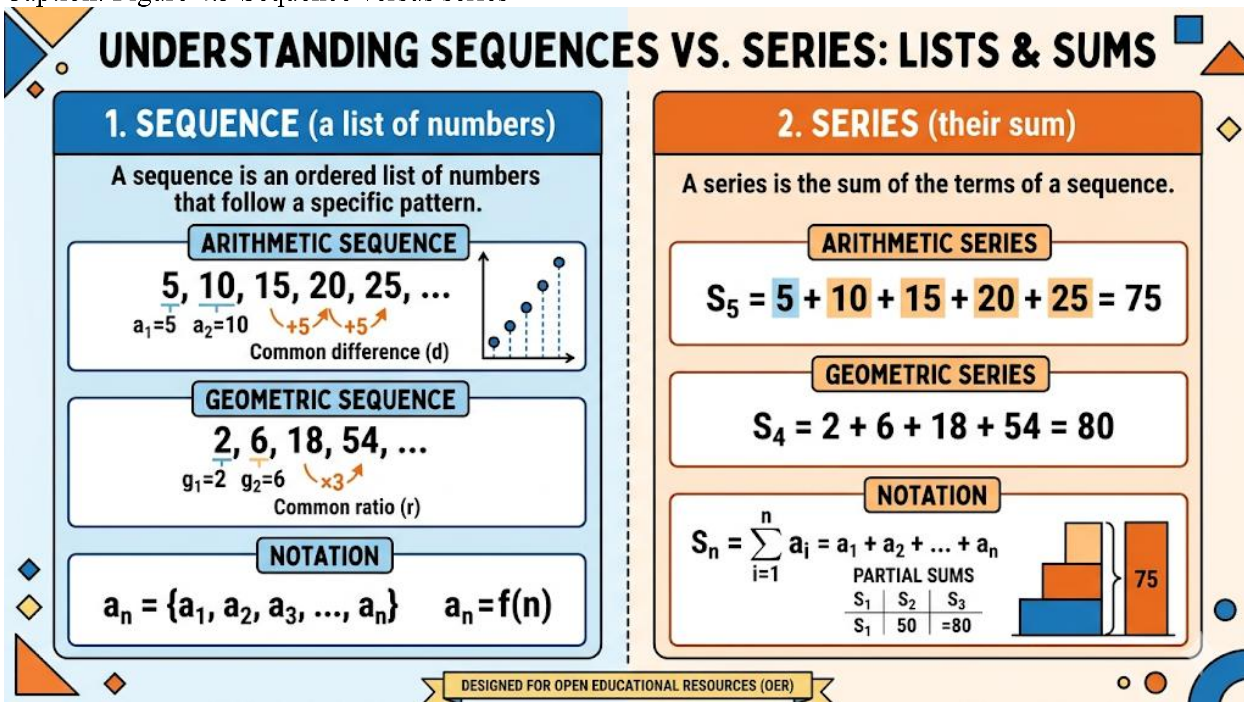
4.2.1 Definition of a series

A **series** is the sum of the terms of a sequence. If the sequence is finite, the series is also finite; if the sequence is infinite, the series may converge to a finite value or diverge. For example, the sequence (2, 4, 6, 8, …) can form a series (2 + 4 + 6 + 8 + …).

Fill in the blank: A series is defined as the ________ of the terms of a sequence.

Diagram showing the difference between a sequence and a series

Caption: Figure 4.3 Sequence versus series



4.2.2 Arithmetic series and formulae

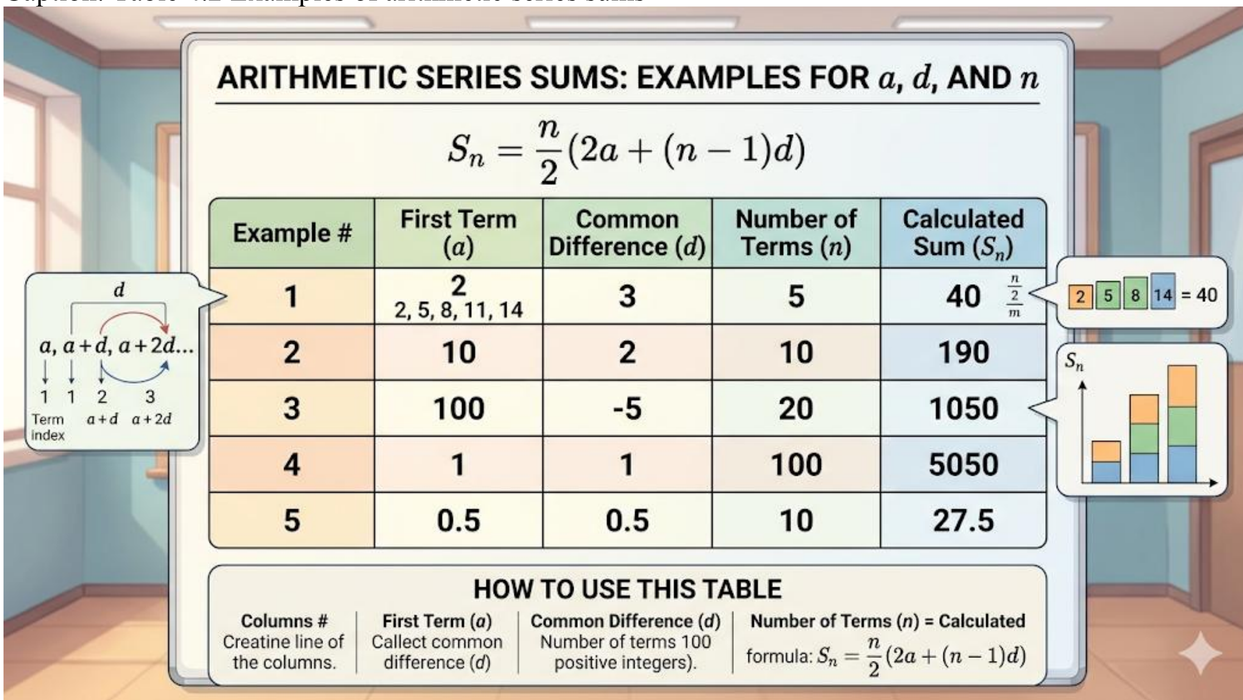
An **arithmetic series** is the sum of terms in an arithmetic sequence. The sum of the first (n) terms is given by:

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

where (a) is the first term, (d) is the common difference, and (n) is the number of terms.

Fill in the blank: The sum of the first (n) terms of an arithmetic series is given by ($S_n = \frac{n}{2}(2a + (n - 1) \times$ ________)).

Caption: Table 4.2 Examples of arithmetic series sums


ARITHMETIC SERIES SUMS: EXAMPLES FOR a, d, AND n

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

Example #	First Term (a)	Common Difference (d)	Number of Terms (n)	Calculated Sum (S_n)
1	2 2, 5, 8, 11, 14	3	5	40
2	10	2	10	190
3	100	-5	20	1050
4	1	1	100	5050
5	0.5	0.5	10	27.5

HOW TO USE THIS TABLE

Columns #	First Term (a)	Common Difference (d)	Number of Terms (n) = Calculated
Creatine line of the columns.	Callect common difference (d)	Number of terms 100 positive integers).	formula: $S_n = \frac{n}{2}(2a + (n-1)d)$

4.2.3 Geometric series and convergence

A **geometric series** is the sum of terms in a geometric sequence. The sum of the first (n) terms is given by:

$$S_n = \frac{a(1 - r^n)}{1 - r}, \quad r \neq 1$$

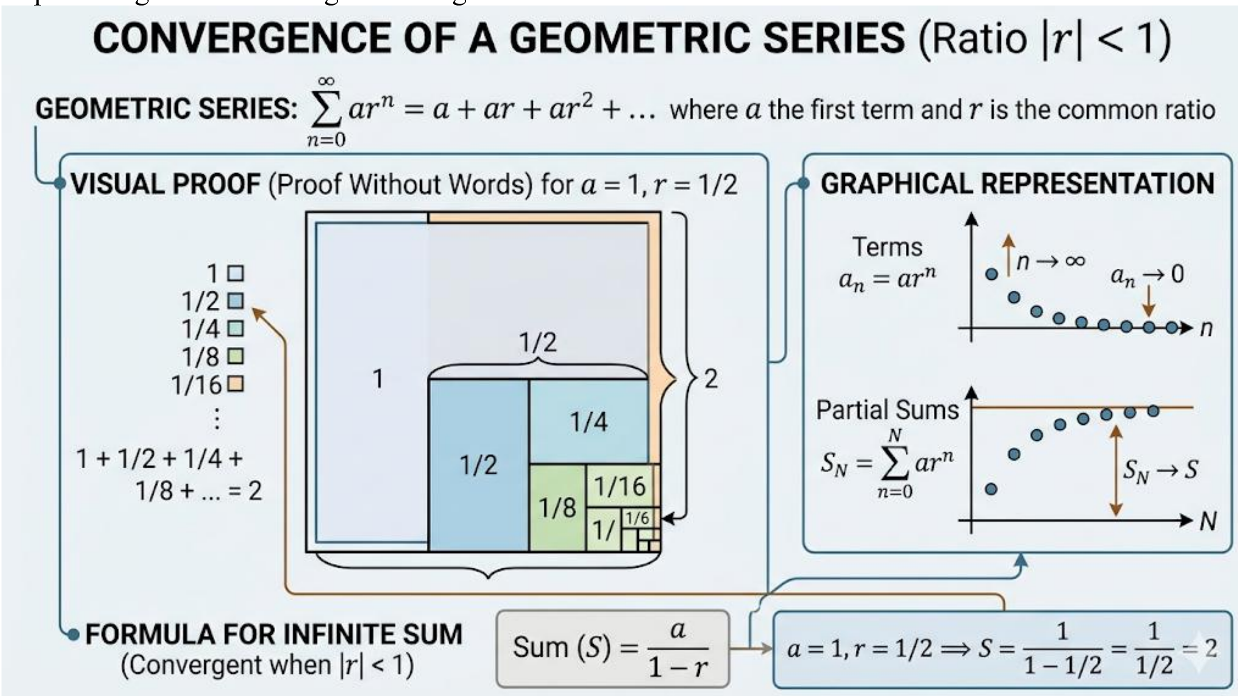
For an infinite geometric series where ($|r| < 1$), the sum converges to:

$$S = \frac{a}{1 - r}$$

Fill in the blank: An infinite geometric series converges only when the absolute value of the common ratio is less than ________.

Diagram showing convergence of a geometric series when ($|r| < 1$)

Caption: Figure 4.4 Convergence of a geometric series



Pilot questions 4.2

1. A series is defined as:
   a. A list of numbers
   b. The sum of terms of a sequence
   c. A difference of terms
   d. A product of terms
2. The sum of the first (n) terms of an arithmetic series is:
   a. ($S_n = \frac{n}{2}(2a + (n - 1)d)$)
   b. ($S_n = ar^{n-1}$)
   c. ($S_n = a + (n - 1)d$)
   d. ($S_n = \frac{a}{1-r}$)
3. The sum of the first (n) terms of a geometric series is:
   a. ($S_n = \frac{a(1 - r^n)}{1 - r}$)
   b. ($S_n = a + (n - 1)d$)
   c. ($S_n = \frac{n}{2}(2a + (n - 1)d)$)
   d. ($S_n = ar^{n-1}$)
4. An infinite geometric series converges when:
   a. ($|r| > 1$)
   b. ($|r| < 1$)
   c. ($r = 1$)
   d. ($r = 0$)
5. The sum of an infinite geometric series is:
   a. ($S = \frac{a}{1-r}$)
   b. ($S = \frac{n}{2}(2a + (n - 1)d)$)
   c. ($S = ar^{n-1}$)
   d. ($S = a + (n - 1)d$)

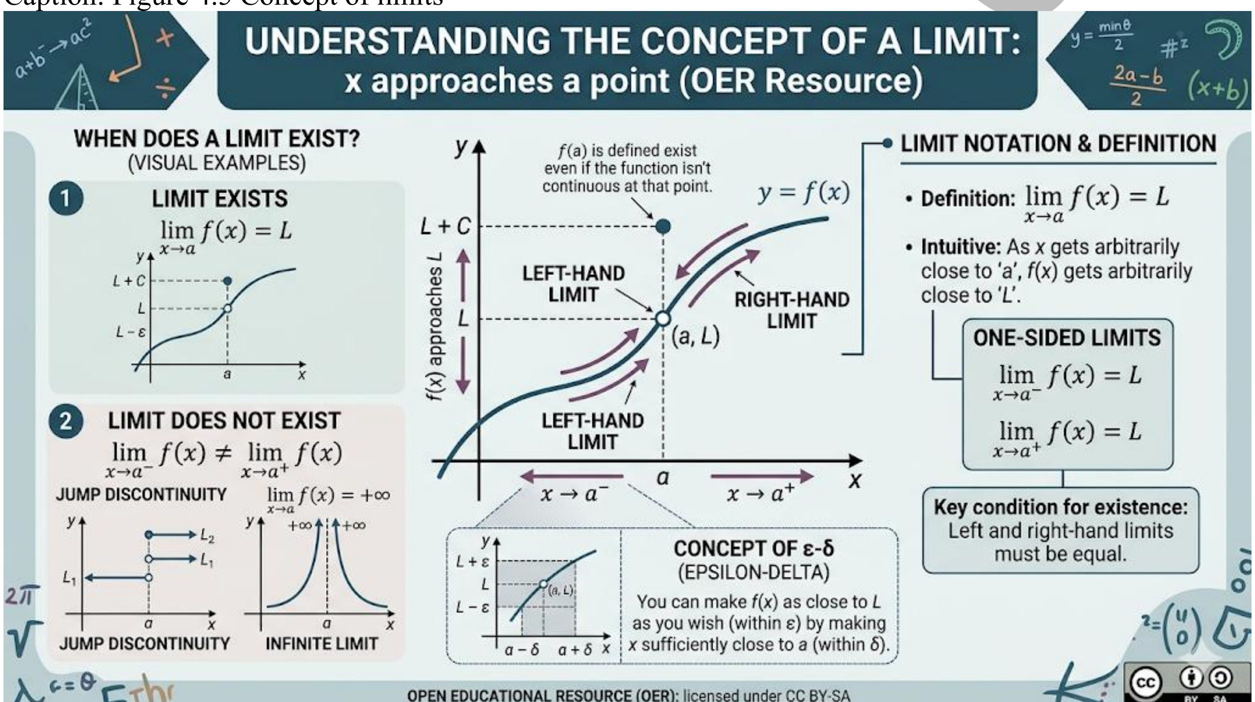
4.3 Limits and Continuity

4.3.1 Concept of limits

A **limit** describes the value that a function approaches as the input approaches a certain point. Limits are fundamental to calculus, as they define continuity, derivatives, and integrals.

Fill in the blank: A limit describes the value that a function approaches as the input approaches a certain ________.

Diagram showing a function approaching a limit as x approaches a point
Caption: Figure 4.5 Concept of limits

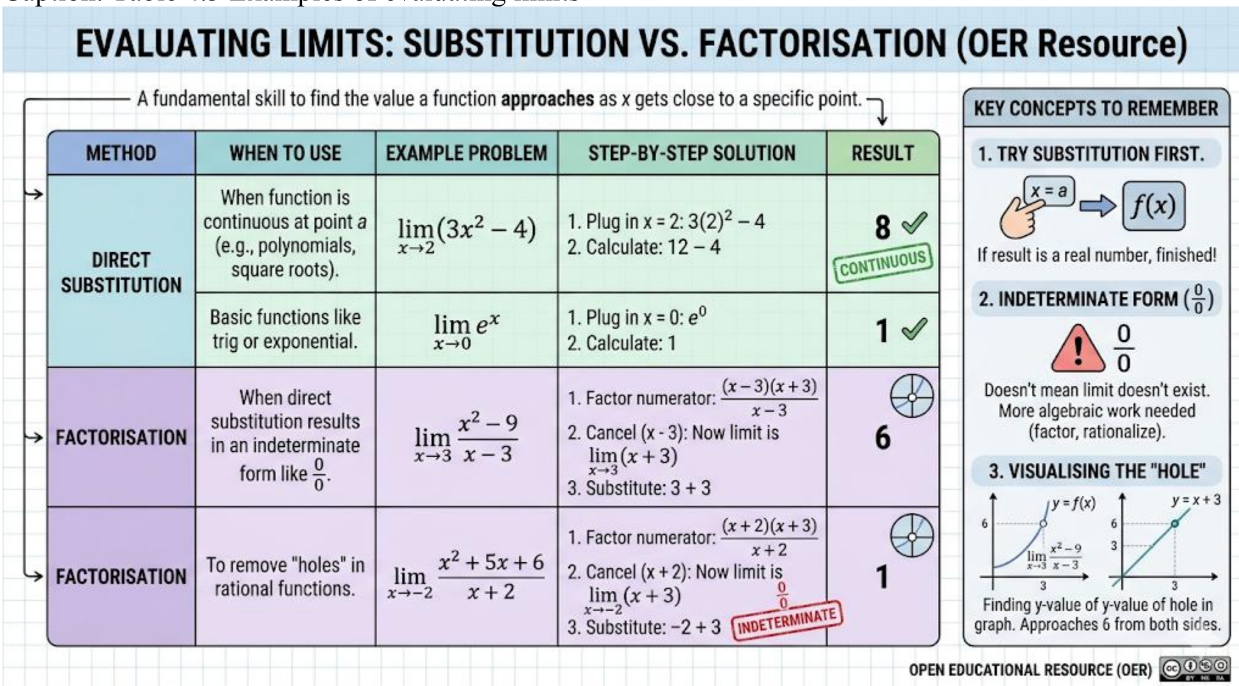


4.3.2 Evaluating limits

Limits can be evaluated using substitution, factorisation, or rationalisation. For example:
[\lim_{x \to 2} (x^2 - 4)/(x - 2) = \lim_{x \to 2} (x + 2) = 4]

Fill in the blank: One method of evaluating limits is ________, where expressions are simplified before substitution.

Caption: Table 4.3 Examples of evaluating limits



EVALUATING LIMITS: SUBSTITUTION VS. FACTORISATION (OER Resource)

A fundamental skill to find the value a function **approaches** as x gets close to a specific point.

METHOD	WHEN TO USE	EXAMPLE PROBLEM	STEP-BY-STEP SOLUTION	RESULT
DIRECT SUBSTITUTION	When function is continuous at point *a* (e.g., polynomials, square roots).	$\lim_{x\to 2}(3x^2-4)$	1. Plug in x = 2: $3(2)^2 - 4$ 2. Calculate: 12 – 4	8 ✓ CONTINUOUS
	Basic functions like trig or exponential.	$\lim_{x\to 0} e^x$	1. Plug in x = 0: e^0 2. Calculate: 1	1 ✓
FACTORISATION	When direct substitution results in an indeterminate form like $\frac{0}{0}$.	$\lim_{x\to 3}\frac{x^2-9}{x-3}$	1. Factor numerator: $\frac{(x-3)(x+3)}{x-3}$ 2. Cancel (x - 3): Now limit is $\lim_{x\to 3}(x+3)$ 3. Substitute: 3 + 3	6
FACTORISATION	To remove "holes" in rational functions.	$\lim_{x\to -2}\frac{x^2+5x+6}{x+2}$	1. Factor numerator: $\frac{(x+2)(x+3)}{x+2}$ 2. Cancel (x + 2): Now limit is $\frac{0}{0}$ $\lim_{x\to -2}(x+3)$ 3. Substitute: −2 + 3 INDETERMINATE	1

KEY CONCEPTS TO REMEMBER

1. TRY SUBSTITUTION FIRST.

x = a → $f(x)$

If result is a real number, finished!

2. INDETERMINATE FORM ($\frac{0}{0}$)

$\frac{0}{0}$

Doesn't mean limit doesn't exist. More algebraic work needed (factor, rationalize).

3. VISUALISING THE "HOLE"

$y = f(x)$ $\lim_{x\to 3}\frac{x^2-9}{x-3}$ $y = x + 3$

Finding y-value of y-value of hole in graph. Approaches 6 from both sides.

OPEN EDUCATIONAL RESOURCE (OER)

4.3.3 Continuity and its importance

A function is **continuous** if its graph has no breaks, jumps, or holes. Continuity ensures that small changes in input produce small changes in output, which is essential in modelling real-world phenomena.

Fill in the blank: A function is continuous if its graph has no ________, jumps, or holes.

Caption: Box 4.1 Importance of continuity in mathematics and business

Field	Core Importance	Practical Application
Mathematics	Differentiability	A function must be continuous to be differentiable. Without continuity, we cannot calculate the instantaneous rate of change (the derivative).
	Intermediate Value Theorem	Ensures that if a continuous function takes two values, it must also take every value in between. This is used to prove the existence of roots/solutions.

Field	Core Importance	Practical Application
Business & Economics	**Price Stability**	Models assume that a small increase in price will lead to a predictable, gradual change in demand rather than an immediate, total collapse of the market.
	Financial Modeling	Continuity allows for the use of Black-Scholes and other calculus-based models to price options and manage risk in "smooth" markets.
Engineering	**System Safety**	Continuous functions model physical phenomena like temperature or velocity, where sudden "jumps" could indicate mechanical failure or physical impossibility.

Pilot questions 4.3

1. A limit describes:
   a. The exact value of a function
   b. The value a function approaches
   c. The slope of a line
   d. The intercept of a line
2. Which method can be used to evaluate limits?
   a. Substitution
   b. Factorisation
   c. Rationalisation
   d. All of the above
3. A function is continuous if:
   a. Its graph has breaks
   b. Its graph has no breaks
   c. Its graph has jumps
   d. Its graph has holes
4. Continuity ensures that:
   a. Small changes in input produce small changes in output
   b. Large changes in input produce small changes in output
   c. Input has no effect on output
   d. Output is always constant
5. Limits are fundamental to:
   a. Geometry
   b. Algebra
   c. Calculus
   d. Trigonometry

4.4 Differentiation and Integration

4.4.1 Basic rules of differentiation

Differentiation is the process of finding the derivative of a function, which represents the rate of change. Basic rules include:

- Power rule: (\frac{d}{dx}(x^n) = nx^{n-1})
- Sum rule: Derivative of a sum is the sum of derivatives.
- Constant rule: Derivative of a constant is zero.

Fill in the blank: The derivative of (x^n) is given by (nx^{n-1}), according to the ________ rule.

Caption: Table 4.4 Basic rules of differentiation

BASIC DIFFERENTIATION RULES: EXAMPLES & APPLICATION (OER RESOURCE)

Learn to evaluate rates of change with fundamental rules.

RULE NAME	FORMULA	STEP-BY-STEP EXAMPLE	APPLICATION
CONSTANT RULE	$\frac{d}{dx}[c] = 0$	$f(x) = 15 \rightarrow f'(x) = 0$	Finding rate of change of a fixed value (e.g., fixed cost).
POWER RULE	$\frac{d}{dx}[x^n] = n \cdot x^{n-1}$	$f(x) = x^4 \rightarrow f'(x) = 4x^3$	Differentiating polynomial terms.
SUM & DIFFERENCE RULE	$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$	$f(x) = x^2 + 5x \rightarrow f'(x) = 2x + 5$	Handling functions with multiple terms.
CONSTANT MULTIPLE RULE	$\frac{d}{dx}[c \cdot f(x)] = c \cdot f'(x)$	$f(x) = 7x^3 \rightarrow f'(x) = 7 \cdot (3x^2) = 21x^2$	Moving a constant out of the derivative.

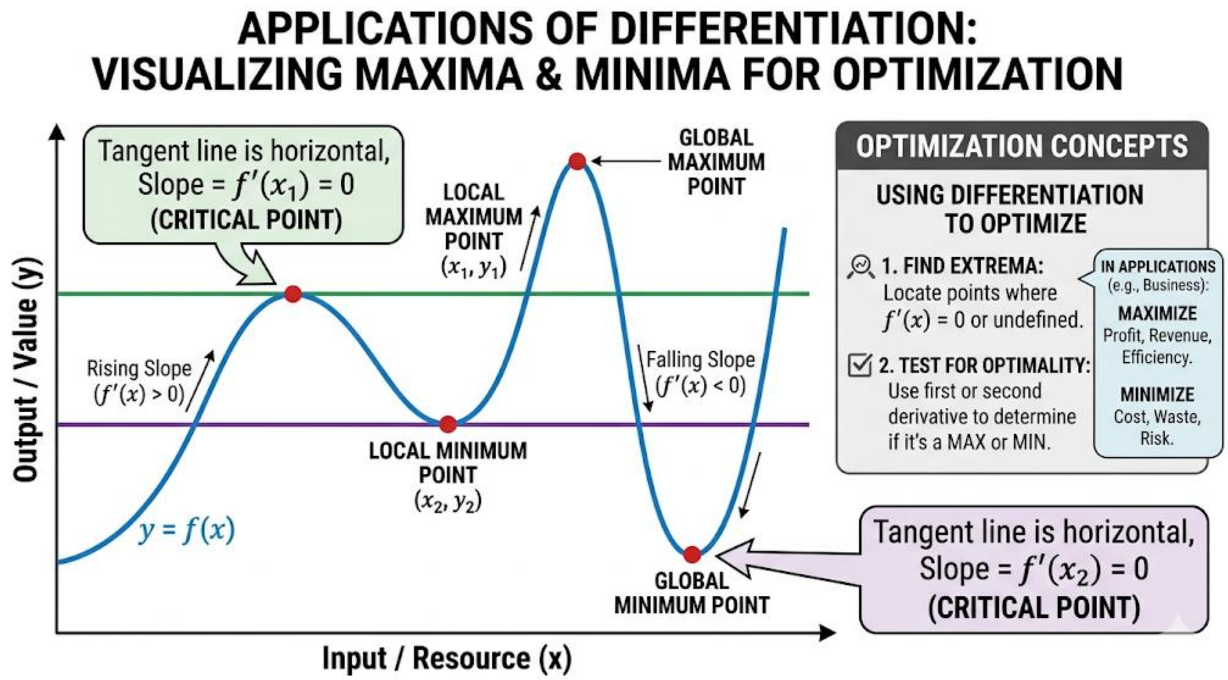
4.4.2 Applications of differentiation (maximum and minimum points)

Differentiation is used to find maximum and minimum points of functions, which are critical in optimisation problems. For example, businesses use differentiation to maximise profit or minimise cost.

Fill in the blank: Differentiation is used in business to maximise ________ or minimise cost.

Diagram showing a curve with maximum and minimum points

Caption: Figure 4.6 Applications of differentiation in optimisation



4.4 Differentiation and Integration (continued)

4.4.3 Basic rules of integration

Integration is the reverse process of differentiation and represents the accumulation of quantities. It is often used to calculate areas under curves or total growth over time. Basic rules include:

- Power rule: ($\int x^n dx = \frac{x^{n+1}}{n+1} + C$), for ($n \neq -1$).
- Sum rule: The integral of a sum is the sum of the integrals.
- Constant rule: The integral of a constant is the constant multiplied by the variable.

Fill in the blank: The integral of (x^n) is given by ($\frac{x^{n+1}}{n+1} + C$), according to the ________ rule.

Caption: Table 4.5 Basic rules of integration

Constant Rule	$\int k\,dx = kx + C$	$\int 5\,dx$	$5x + C$
Power Rule ($n \neq -1$)	$\int x^n\,dx = x^{n+1}/(n+1) + C$	$\int x^3\,dx$	$x^4/4 + C$
Power Rule (fractional exponent)	$\int x^n\,dx = x^{n+1}/(n+1) + C$	$\int \sqrt{x}\,dx = \int x^{(1/2)}\,dx$	$(2/3)x^{(3/2)} + C$

| **Power Rule** (negative exponent) | $\int x^n\,dx = x^{n+1}/(n+1) + C$ | $\int 1/x^2\,dx = \int x^{(-2)}\,dx$ | $-1/x + C$ |

| **Constant Multiple Rule** | $\int k\cdot f(x)\,dx = k\cdot\int f(x)\,dx$ | $\int 6x^2\,dx$ | $6\cdot(x^3/3) + C = 2x^3 + C$ |

| **Sum Rule** | ∫ \[f(x) + g(x)] dx = ∫ f(x) dx + ∫ g(x) dx | $\int (x + x^2)\,dx$ | $x^2/2 + x^3/3 + C$ |

| **Difference Rule** | ∫ \[f(x) - g(x)] dx = ∫ f(x) dx - ∫ g(x) dx | $\int (x - x^2)\,dx$ | $x^2/2 - x^3/3 + C$ |

| **Combined Rules** | Apply multiple rules together | $\int (4x^3 - 6x^2 + 2x + 5)\,dx$ | $x^4 - 2x^3 + x^2 + 5x + C$ |

4.4.4 Applications of integration in mathematics and business

Integration is widely applied in mathematics to calculate areas, volumes, and accumulated quantities. In business, integration can be used to determine total revenue over time, cumulative costs, or growth trends. For example, integrating a demand function can provide total demand over a given period.

Fill in the blank: Integration can be used in business to calculate total ________ over time.

Caption: Box 4.2 Applications of integration in mathematics and business

Applications of Integration in Mathematics & Business

MATHEMATICAL APPLICATIONS

Area & Volume Calculations
- Areas of irregular shapes and regions between curves
- Volumes of solids with curved sides (revolution, shells)
- Surface areas of 3D objects from rotated curves

Physics & Engineering
- Center of Mass: Balance points of objects and systems
- Work Calculations: $W = \int F(x)\,dx$ (springs, lifting)
- Arc Length: Measuring curved paths (rockets, roads)
- Fluid Pressure: Force on submerged surfaces

Motion Analysis
- Distance from velocity: $d = \int v(t)\,dt$
- Displacement and total distance in kinematics

BUSINESS & ECONOMIC APPLICATIONS

Cost & Revenue Analysis
- Total Cost: $C(x) = \int MC(x)\,dx$ (from marginal cost)
- Total Revenue: $R(x) = \int MR(x)\,dx$ (from marginal revenue)
- Profit Maximization: Areas between MC and MR curves

Consumer & Producer Surplus
- Economic welfare: Areas between demand/supply curves
- Benefits from market transactions

Capital & Resource Management
- Accumulated investment from varying rates
- Asset depreciation calculations
- Inventory levels from production/sales rates
- Fuel consumption analysis for fleets

KEY EXAMPLES

Context	Application	Integration Use
Architecture	Irregular land plots	Area under curves
Manufacturing	Cell phone production	$\int(0.01x^2 - 3x + 229)dx$
Amusement Parks	Ticket sales profit	Area between MC & MR curves
Urban Planning	Population growth	Integrating growth rate functions

CORE PRINCIPLE: Integration reverses differentiation to find totals from rates—whether total area from boundary curves, total distance from velocity, or total cost from marginal cost. This "summing up" process makes it indispensable for solving real-world problems with continuous change.

Pilot questions 4.4

1. The derivative of (x^5) is:
   a. (5x^4)
   b. (x^4)
   c. (6x^5)
   d. (5x^5)
2. Differentiation is used in business to:
   a. Calculate total revenue
   b. Maximise profit or minimise cost
   c. Determine continuity
   d. Evaluate limits
3. The integral of (x^3) is:
   a. (\frac{x^4}{4} + C)
   b. (3x^2)
   c. (\frac{x^3}{3} + C)
   d. (x^4)
4. Integration is used in mathematics to calculate:
   a. Slopes of curves
   b. Areas under curves
   c. Maximum points
   d. Continuity
5. In business, integration can be used to calculate:
   a. Break-even points
   b. Total revenue over time
   c. Common differences
   d. Market ratios

Series Summary

This Series has introduced you to sequences, series, and calculus, which are fundamental tools for analysing patterns, cumulative totals, and continuous change. We began with sequences, distinguishing between arithmetic and geometric types and learning how to calculate their terms. We then moved on to series, exploring summation techniques for arithmetic and geometric series and examining convergence in infinite series. Following that, we studied limits and continuity, which form the foundation of calculus, and finally we concluded with differentiation and integration, focusing on their rules and applications in mathematics and business.

By completing this Series, you should now be able to define and calculate terms of sequences, apply summation techniques to series, evaluate limits and continuity, and perform differentiation and integration with practical applications. These skills directly align with the Series Learning Outcomes and provide a strong foundation for advanced mathematical problem-solving and business modelling.

Glossary of Terms

- **Arithmetic sequence:** A sequence where each term is obtained by adding a fixed number (common difference) to the previous term.
- **Arithmetic series:** The sum of terms in an arithmetic sequence.
- **Common difference (d):** The fixed number added to each term in an arithmetic sequence.
- **Common ratio (r):** The fixed number multiplied to each term in a geometric sequence.
- **Continuity:** A property of a function where its graph has no breaks, jumps, or holes.
- **Derivative:** The rate of change of a function, found through differentiation.
- **Geometric sequence:** A sequence where each term is obtained by multiplying the previous term by a fixed number.
- **Geometric series:** The sum of terms in a geometric sequence.
- **Integration:** The reverse of differentiation, representing accumulation or area under a curve.
- **Limit:** The value a function approaches as the input approaches a certain point.
- **Sequence:** An ordered list of numbers following a specific rule or pattern.
- **Summation:** The process of adding terms of a sequence to form a series.

Concept Check Questions 4

Objective questions

1. Which of the following is an arithmetic sequence? (Linked to SLO 4.1)
2. The nth term of an arithmetic sequence is given by: (Linked to SLO 4.2)
3. The sum of the first (n) terms of a geometric series is: (Linked to SLO 4.4)
4. A limit describes: (Linked to SLO 4.5)
5. Differentiation is used in business to: (Linked to SLO 4.7)

Short-answer questions

6. Define a geometric sequence and give one example. (Linked to SLO 4.1)
7. State the formula for the sum of the first (n) terms of an arithmetic series. (Linked to SLO 4.3)
8. Explain what continuity means in calculus. (Linked to SLO 4.6)

Long-answer questions

9. Demonstrate how to evaluate an infinite geometric series and explain its convergence condition. (Linked to SLO 4.4)
10. Analyse how differentiation and integration can be applied in business mathematics, using examples such as profit maximisation and cumulative revenue. (Linked to SLO 4.9)

Answers to Concept Check Questions 4

Objective questions

1. 2, 4, 6, 8, ...
2. ($T_n = a + (n - 1)d$)
3. ($S_n = \frac{a(1 - r^n)}{1 - r}$)
4. The value a function approaches as the input approaches a point
5. Maximise profit or minimise cost

Short-answer questions

6. A geometric sequence is one where each term is obtained by multiplying the previous term by a fixed number called the common ratio. Example: 2, 4, 8, 16, ... with common ratio 2.
7. The sum of the first (n) terms of an arithmetic series is ($S_n = \frac{n}{2}(2a + (n - 1)d)$).
8. Continuity means that a function's graph has no breaks, jumps, or holes, ensuring smooth behaviour of the function.

Long-answer questions

9. *Basic response:* An infinite geometric series converges when ($|r| < 1$). Its sum is given by ($S = \frac{a}{1 - r}$).
   Value-added response: This convergence condition is important in modelling scenarios such as depreciation or discounting in finance, where values diminish over time and converge to a finite total.
10. *Basic response:* Differentiation can be used to find maximum profit or minimum cost, while integration can calculate total revenue over time.
   Value-added response: In business, differentiation helps optimise production levels, while integration supports forecasting cumulative demand or revenue. These applications enable managers to make strategic decisions based on mathematical analysis.

Answers to Pilot Questions [Series 4]

Part 4.1: Sequences and Their Properties

1. a
2. b
3. a
4. b
5. b

Part 4.2: Series and Summation Techniques

1. b
2. a
3. a
4. b

5. a

Part 4.3: Limits and Continuity

1. b
2. d
3. b
4. a
5. c

Part 4.4: Differentiation and Integration

1. a
2. b
3. a
4. b
5. b

References and Attributions [Series 4]

- Suggested illustration sources:
    - Figure 4.1 Types of sequences: “arithmetic and geometric sequences diagram OER”
    - Table 4.1 Arithmetic sequence examples: “arithmetic sequence nth term examples OER”
    - Figure 4.2 Geometric sequence representation: “geometric sequence examples diagram OER”
    - Figure 4.3 Sequence versus series: “sequence vs series diagram OER”
    - Table 4.2 Arithmetic series sums: “arithmetic series sum examples OER”
    - Figure 4.4 Convergence of a geometric series: “geometric series convergence diagram OER”
    - Figure 4.5 Concept of limits: “limit concept diagram OER”
    - Table 4.3 Evaluating limits: “evaluating limits examples OER”
    - Box 4.1 Importance of continuity: “continuity importance mathematics OER”
    - Table 4.4 Differentiation rules: “basic differentiation rules examples OER”
    - Figure 4.6 Differentiation in optimisation: “applications of differentiation optimisation diagram OER”
    - Table 4.5 Integration rules: “basic integration rules examples OER”
    - Box 4.2 Applications of integration: “applications of integration in business OER”
- AI prompt suggestions were provided for each illustration to support future generation using AI tools.
- References:
    - Open Educational Resources (OER) in sequences, series, and calculus.
    - Standard texts in business mathematics covering sequences, series, limits, differentiation, and integration.

Series 5: Probability and Statistics

- **Part 5.1: Fundamentals of Probability**
  - Sub-Part 5.1.1: Definition and basic concepts of probability
  - Sub-Part 5.1.2: Rules of probability (addition and multiplication)
  - Sub-Part 5.1.3: Conditional probability and independence
- **Part 5.2: Random Variables and Probability Distributions**
  - Sub-Part 5.2.1: Definition of random variables
  - Sub-Part 5.2.2: Discrete probability distributions (binomial, Poisson)
  - Sub-Part 5.2.3: Continuous probability distributions (normal distribution)
- **Part 5.3: Fundamentals of Statistics**
  - Sub-Part 5.3.1: Definition and importance of statistics
  - Sub-Part 5.3.2: Descriptive statistics (mean, median, mode, variance, standard deviation)
  - Sub-Part 5.3.3: Data presentation (tables, charts, graphs)
- **Part 5.4: Statistical Inference and Applications**
  - Sub-Part 5.4.1: Sampling and sampling distributions
  - Sub-Part 5.4.2: Hypothesis testing
  - Sub-Part 5.4.3: Applications of probability and statistics in business and decision-making

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Introduction

Background
Probability and statistics are essential tools for analysing uncertainty and making informed decisions. Probability helps us quantify the likelihood of events, while statistics provides methods for collecting, summarising, and interpreting data. Together, they form the backbone of modern business analytics, risk management, and scientific research.

Series Aim
The aim of this Series is to introduce you to the principles of probability and statistics, and to equip you with the skills needed to analyse data, evaluate uncertainty, and apply statistical methods in mathematics and business contexts.

Series Synopsis
We shall commence this Series by exploring the fundamentals of probability, including its definition, rules, and conditional concepts. Next, we will move on to random variables and probability distributions, examining both discrete and continuous types. Following that, we will study the fundamentals of statistics, focusing on descriptive measures and data presentation. Finally, we will conclude with statistical inference, covering sampling, hypothesis testing, and practical applications in business decision-making.

Series Learning Outcomes

Upon completing this Series, you should be able to:

- **SLO 5.1** define probability and explain its basic concepts,
- **SLO 5.2** apply rules of probability including addition and multiplication,
- **SLO 5.3** demonstrate conditional probability and independence,
- **SLO 5.4** define random variables and distinguish between discrete and continuous types,
- **SLO 5.5** calculate probabilities using binomial, Poisson, and normal distributions,
- **SLO 5.6** define statistics and explain its importance,
- **SLO 5.7** compute descriptive statistics such as mean, median, mode, variance, and standard deviation,
- **SLO 5.8** present data effectively using tables, charts, and graphs,
- **SLO 5.9** apply sampling techniques and explain sampling distributions,
- **SLO 5.10** perform hypothesis testing, and
- **SLO 5.11** evaluate applications of probability and statistics in business and decision-making.

5.1 Fundamentals of Probability

5.1.1 Definition and basic concepts of probability

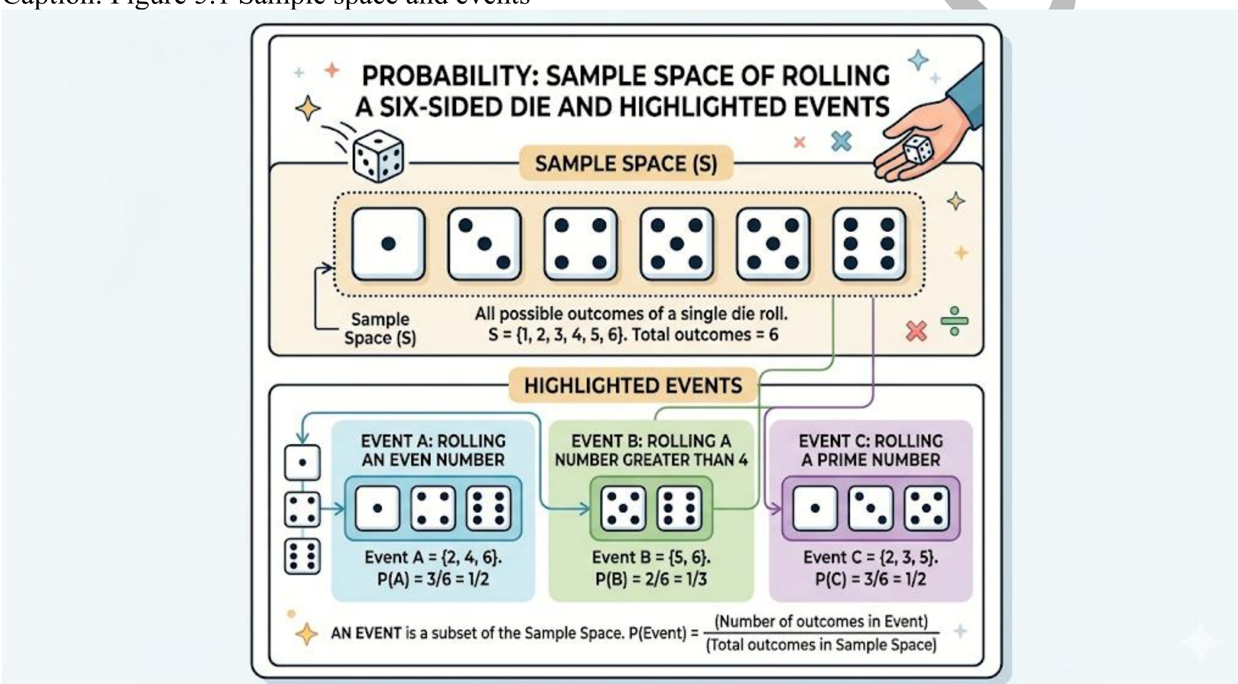
Probability is the measure of how likely an event is to occur. It is expressed as a number between 0 and 1, where 0 means the event is impossible and 1 means the event is certain. For example, the probability of flipping a fair coin and getting heads is 0.5.

Key concepts include:

- **Experiment:** A process that leads to an outcome (e.g., rolling a die).
- **Sample space:** The set of all possible outcomes (e.g., {1, 2, 3, 4, 5, 6}).
- **Event:** A subset of the sample space (e.g., rolling an even number).

Fill in the blank: The set of all possible outcomes of an experiment is called the ________.

Diagram showing sample space of rolling a die and highlighting events
Caption: Figure 5.1 Sample space and events



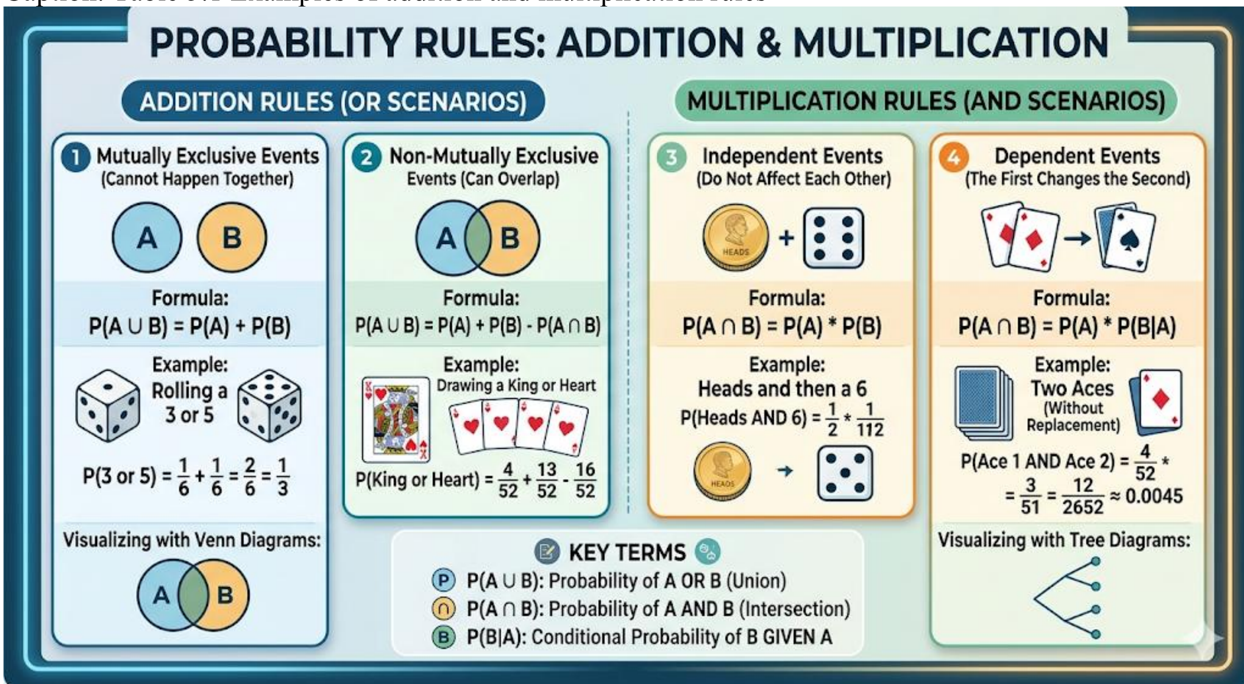
5.1.2 Rules of probability (addition and multiplication)

Probability rules help us calculate the likelihood of combined events:

- **Addition rule:** For mutually exclusive events, ($P(A \text{ or } B) = P(A) + P(B)$).
- **Multiplication rule:** For independent events, ($P(A \text{ and } B) = P(A) \times P(B)$).

Fill in the blank: For independent events, the probability of both occurring is found using the ________ rule.

Caption: Table 5.1 Examples of addition and multiplication rules



5.1.3 Conditional probability and independence

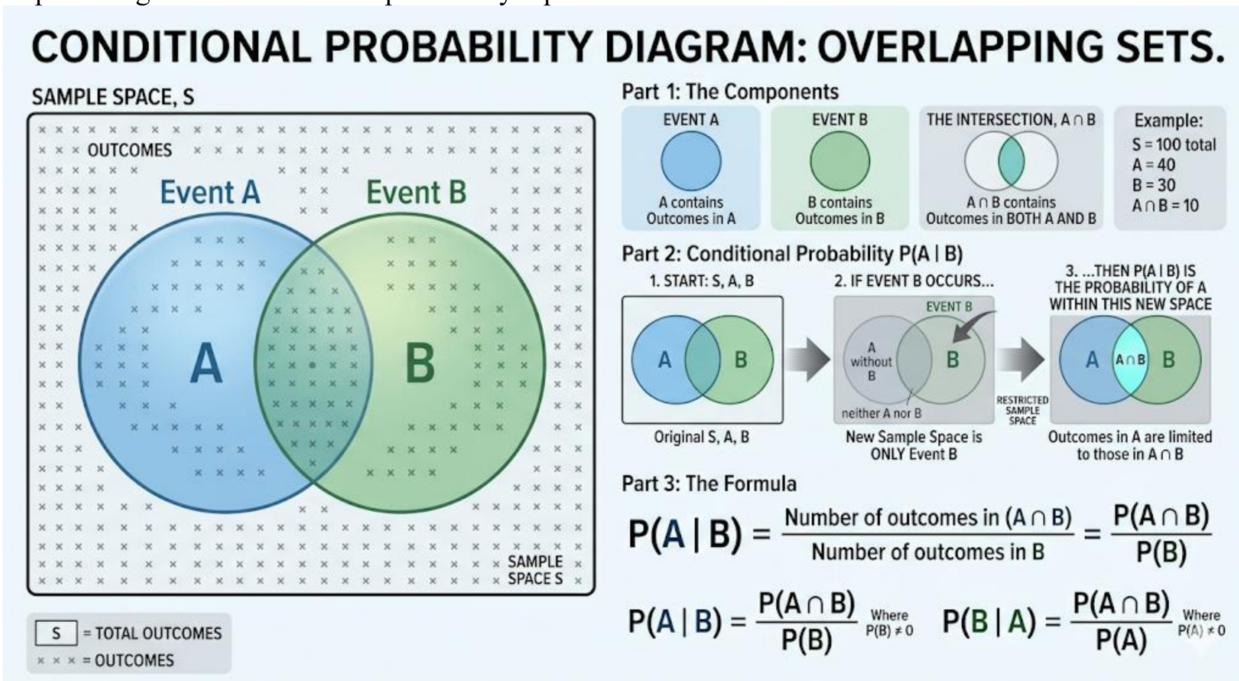
Conditional probability is the probability of an event occurring given that another event has already occurred. It is written as (P(A|B)). Events are **independent** if the occurrence of one does not affect the probability of the other.

For example, the probability of drawing a red card given that the card is a heart is (P(\text{Red}|\text{Heart}) = 1).

Fill in the blank: Conditional probability is written as (P(A|B)), meaning the probability of A occurring given that ________ has occurred.

Diagram showing conditional probability with overlapping sets

Caption: Figure 5.2 Conditional probability representation



Pilot questions 5.1

1. Probability is expressed as a number between:
   a. -1 and 1
   b. 0 and 1
   c. 1 and 10
   d. 0 and 100
2. The set of all possible outcomes of an experiment is called:
   a. Event
   b. Sample space
   c. Probability
   d. Outcome
3. For mutually exclusive events, the probability of A or B is:
   a. (P(A) \times P(B))
   b. (P(A) + P(B))
   c. (P(A) - P(B))
   d. (P(A|B))
4. For independent events, the probability of A and B is:
   a. (P(A) + P(B))
   b. (P(A) \times P(B))
   c. (P(A) - P(B))
   d. (P(A|B))
5. Conditional probability is written as:
   a. (P(A + B))
   b. (P(A \times B))
   c. (P(A|B))
   d. (P(A - B))

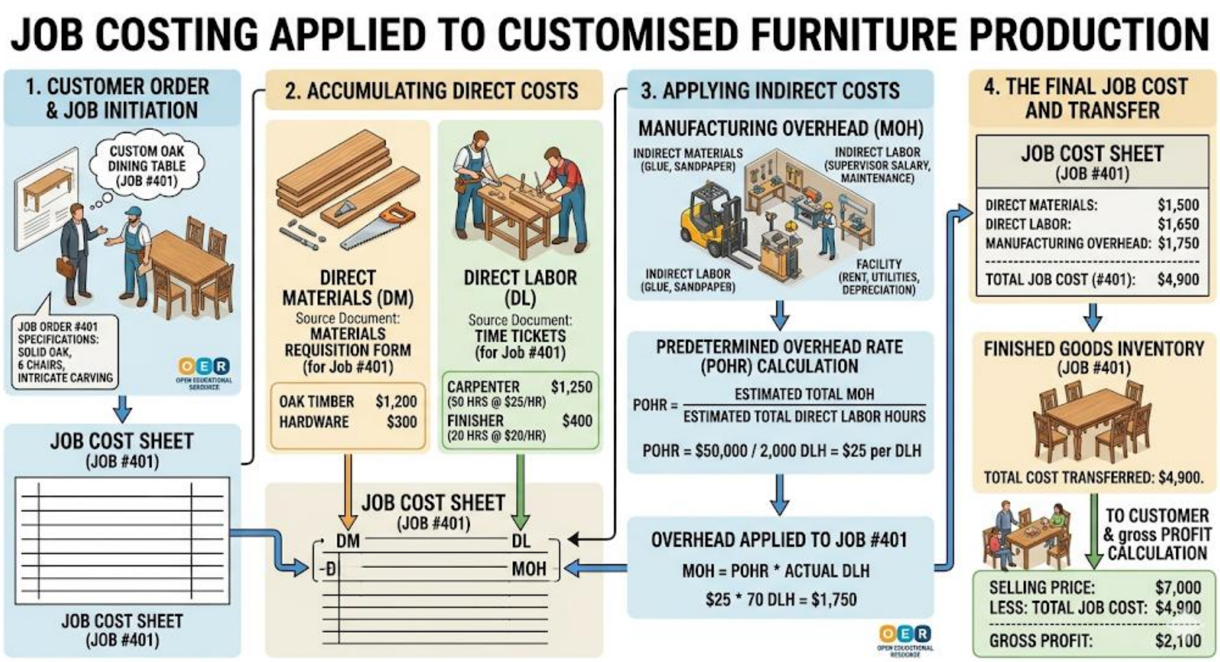
5.2 Specific Costing Techniques

5.2.1 Job costing

Job costing is a method used when products are manufactured based on specific customer orders. Costs are traced and accumulated for each job separately. This method is common in industries such as construction, printing, and customised furniture.

Fill in the blank: Job costing is appropriate when products are manufactured based on specific __________ orders.

Diagram showing job costing applied to customised furniture production.
Caption: Figure 5.2 Job costing in customised production



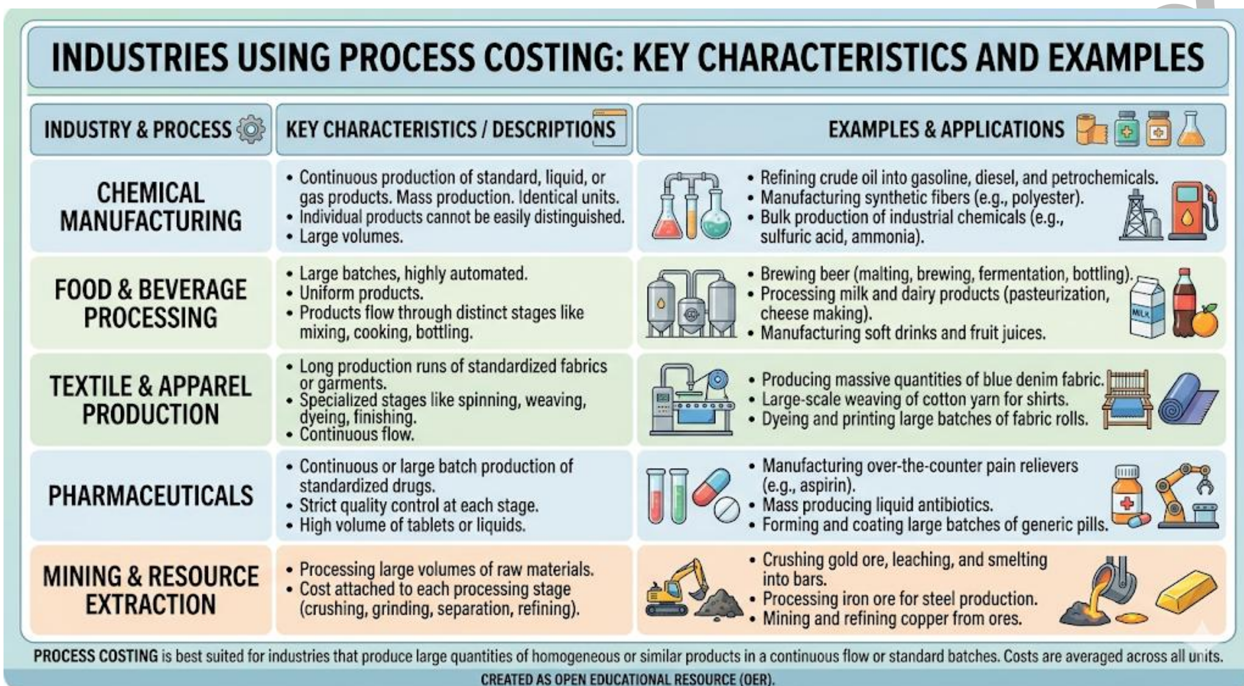
5.2.2 Process costing

Process costing is used when products are produced continuously and are indistinguishable from one another. Costs are accumulated for each process or department and then averaged over all units produced. Industries such as chemicals, food processing, and textiles commonly use process costing.

Fill in the blank: Process costing is suitable for industries where production is __________ and continuous.

Caption: Table 5.2 Examples of industries using process costing

Industry	Description	Example
Chemical	Continuous production	Fertiliser manufacturing
Food processing	Homogeneous products	Biscuit production
Textiles	Mass production	Cotton fabric

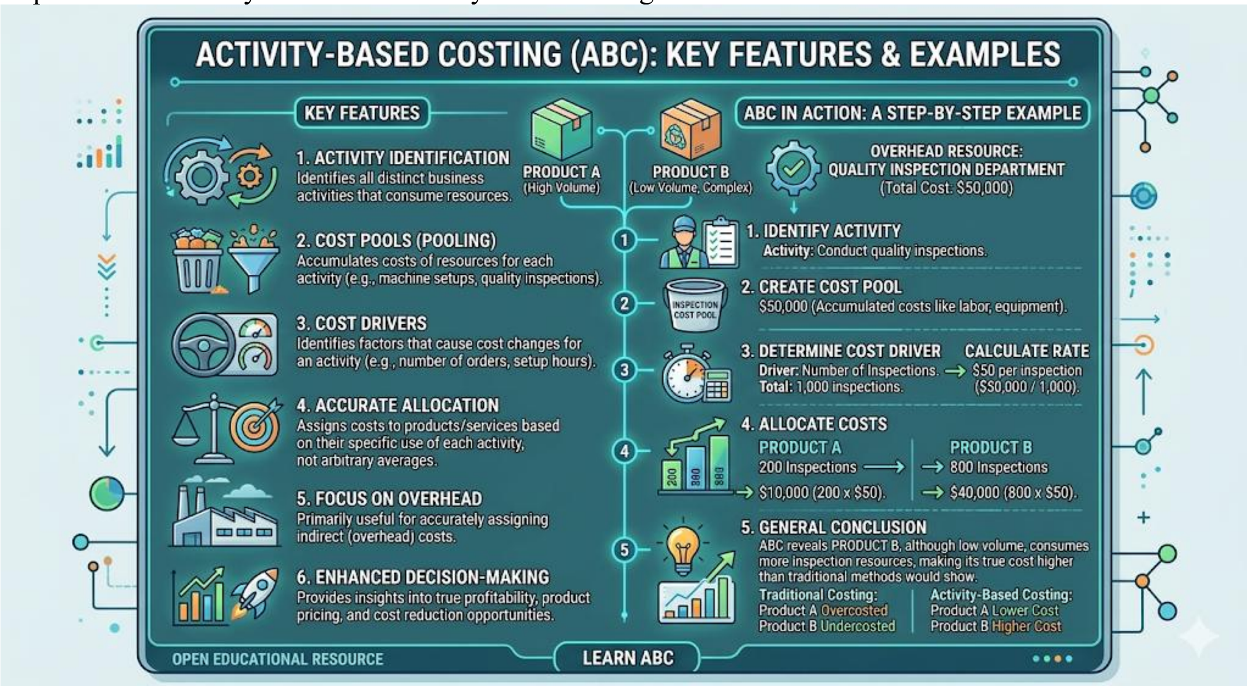


5.2.3 Activity-based costing (ABC)

Activity-based costing (ABC) assigns overhead costs to products based on the activities that drive those costs. It provides more accurate cost information in environments with high overheads and diverse products. ABC is particularly useful in modern manufacturing and service industries.

Fill in the blank: Activity-based costing assigns overhead costs to products based on the __________ that drive those costs.

Caption: Box 5.1 Key features of activity-based costing



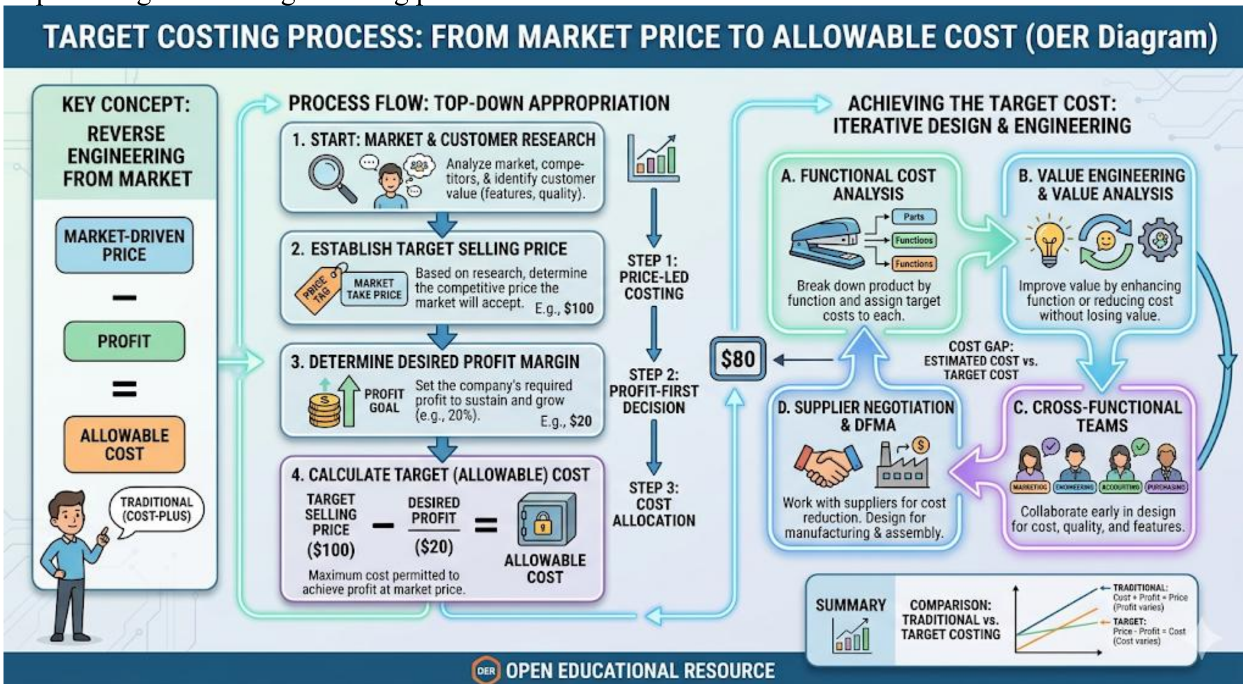
5.2.4 Target costing

Target costing begins with a competitive market price and desired profit margin, then works backwards to determine the allowable cost of production. It is widely used in industries where pricing pressure is high, such as consumer electronics and automobiles.

Fill in the blank: Target costing begins with a competitive __________ price and desired profit margin.

Diagram showing target costing process from market price to allowable cost.

Caption: Figure 5.3 Target costing process



Pilot questions 5.2

1. What is job costing and in which industries is it commonly used?
2. Fill in the blank: Process costing is suitable for industries where production is __________ and continuous.
3. State one advantage of activity-based costing.
4. Fill in the blank: Target costing begins with a competitive __________ price and desired profit margin.
5. Give one example of an industry where process costing is applied.

Series 5 Summary

This Series has introduced you to **costing methods**, which are essential for determining the cost of producing goods and services and for supporting managerial decision-making. We began with the meaning and importance of costing methods, highlighting their role in pricing, budgeting, and performance evaluation. We then examined the criteria for selecting appropriate costing methods, considering product nature, production process, cost structure, and managerial objectives. Following that, we explored specific costing techniques such as job costing, process costing, activity-based costing (ABC), and target costing, each suited to different business contexts.

By completing this Series, you should now be able to define costing methods, identify criteria for their selection, and apply specific techniques to various production and business scenarios. These skills align with the Series Learning Outcomes and provide a foundation for effective cost management and strategic decision-making in business.

Glossary of Terms

- **Activity-based costing (ABC):** A costing method that assigns overhead costs to products based on activities that drive those costs.
- **Costing methods:** Systematic approaches used to ascertain the cost of producing goods or services.
- **Job costing:** A method used for customised products, where costs are traced to specific jobs or orders.
- **Managerial objectives:** Goals such as pricing, control, or profitability that influence the choice of costing method.
- **Process costing:** A method used for continuous production of homogeneous products, where costs are averaged across units.
- **Target costing:** A method that begins with a competitive market price and desired profit margin to determine allowable production costs.
- **Cost structure:** The proportion of direct and indirect costs in production, influencing the choice of costing method.
- **Sample space (from probability context earlier):** The set of all possible outcomes of an experiment, included here for continuity with Series 5's broader scope.

Concept Check Questions 5

Objective questions

1. Costing methods are systematic approaches used to ascertain the ________ of producing goods or services.
2. Job costing is most suitable for:
   a. Continuous production
   b. Customised products
   c. Homogeneous products
   d. Mass production
3. Process costing is appropriate when production is:
   a. Customised
   b. Continuous and homogeneous
   c. Irregular
   d. Small-scale

4. Activity-based costing assigns overhead costs based on:
   a. Direct labour hours
   b. Activities that drive costs
   c. Machine hours
   d. Units produced
5. Target costing begins with a competitive ________ price and desired profit margin.

Short-answer questions

6. State two criteria for selecting costing methods.
7. Define activity-based costing and explain its importance.
8. Give one example of an industry where process costing is applied.

Long-answer questions

9. Compare job costing and process costing, highlighting their differences and suitable applications.
10. Analyse how target costing can be applied in competitive markets, using examples from consumer goods industries.

Answers to Concept Check Questions 5

Objective questions

1. Cost
2. b. Customised products
3. b. Continuous and homogeneous
4. b. Activities that drive costs
5. Market

Short-answer questions

6. Criteria include: nature of the product, production process, cost structure, and managerial objectives.
7. Activity-based costing assigns overhead costs to products based on activities that drive those costs. It is important because it provides more accurate cost information in environments with high overheads.
8. Example: Chemical industry, such as fertiliser manufacturing.

Long-answer questions

9. *Basic response:* Job costing is used for customised products, while process costing is used for continuous production of homogeneous products.
   Value-added response: Job costing suits industries like construction and printing, where each job is unique. Process costing suits industries like chemicals and textiles, where products are indistinguishable. The choice depends on production type and managerial needs.

10. *Basic response:* Target costing starts with a market price and desired profit margin, then determines allowable costs.
    Value-added response: In consumer electronics, companies use target costing to design products that meet customer expectations at competitive prices. This ensures profitability while maintaining market competitiveness.

Answers to Pilot Questions [Series 5]

Part 5.1: Introduction to Costing Methods

1. Costing methods are systematic approaches used to ascertain the cost of producing goods or services.
2. Cost
3. Nature of product, production process (others include cost structure and managerial objectives).
4. Job costing.
5. Product.

Part 5.2: Specific Costing Techniques

1. Job costing is a method used for customised products, common in industries like construction and printing.
2. Continuous.
3. Provides more accurate overhead allocation.
4. Market.
5. Food processing industry (e.g., biscuit production).

References and Attributions [Series 5]

- Suggested illustration sources:

- Figure 5.1 Importance of costing methods: “importance of costing methods diagram OER”
- Table 5.1 Criteria for selecting costing methods: “criteria for selecting costing methods table OER”
- Figure 5.2 Job costing in customised production: “job costing customised production diagram OER”
- Table 5.2 Process costing industries: “process costing industries examples OER”
- Box 5.1 Features of activity-based costing: “activity-based costing features OER”
- Figure 5.3 Target costing process: “target costing process diagram OER”

- AI prompt suggestions were provided for each illustration to support future generation using AI tools.
- References:
  - Open Educational Resources (OER) in cost accounting and management accounting.
  - Standard texts in managerial accounting covering costing methods and applications.